Set No. 1

II B.Tech II Semester Regular Examinations, Apr/May 2006
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int\limits_{1}^{1} \frac{x^2 dx}{\sqrt{1-x^5}}$ interms of β function.

   (b) Prove that $\int\limits_{0}^{1} (1-x^n)^{1/n} dx = \frac{1}{n} \frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{2\Gamma(2/n)}$

   (c) Prove that $\Gamma\left(\frac{1}{n}\right) \Gamma\left(\frac{2}{n}\right) \Gamma\left(\frac{3}{n}\right)$ ....$\Gamma\left(\frac{n-1}{n}\right) = \frac{(2\prod)^{\frac{n-1}{2}}}{n^{1/2}}$ [5+5+6]

2. (a) Show that 4 J_n''(x)=J_{n-2}(x)-2J_n(x)+J_{n+2}(x).

   (b) Prove that P_{n+1}'+p_n' =P_0 + 3P_1 + 5P_2 +. . . (2n+1) P_n. [8+8]

3. (a) Show that the function u=2 log (x^2+ y^2) is harmonic and find its harmonic conjugate

   (b) Separate the real and imaginary parts of tan hz [8+8]

4. (a) Evaluate $\int\limits_{c} \frac{e^z\ dz}{(z^2+\prod^2)^2}$ where c is | z | = 4 using Cauchy's integral formula.

   (b) Evaluate $\int\limits_{1-i}^{2+i} (2x+iy+1)dz$ along the straight line joining (1, -i) and (2,i)

   (c) Evaluate $\int\limits_{c} \frac{dz}{z^3(z+4)dz}$ where c is | z | = 2 using Cauchy's integral formula

   [5+5+6]

5. (a) For the function f(z)= $\frac{2z^3+1}{z\,(z+1)}$ find Taylor's series valid in a neighbourhood of z=1

   (b) Find Laurent's series for f(z) = $\frac{1}{z^2\,(1-z)}$ and find the region of convergence

   [8+8]

6. (a) Find the poles and residues at each pole $\frac{ze^z}{(z-1)^3}$

   (b) Evaluate $\int\limits_{C} \frac{2e^z dz}{z(z-3)}$ where C is |z| = 2 by residue theorem. [8+8]

7. (a) Evaluate $\int\limits_{0}^{2\pi} \frac{\sin^2\theta\, d\theta}{6+3\cos\theta}$ using residue theorem.

(b) Evaluate $\int_0^{\alpha} \frac{dx}{x^6+1}$ using residue theorem. [8+8]

8. (a) Find and plot the rectangular region 0≤x≤1; 0≤y≤z, under the transformation w =$\sqrt{}$ 2 e$^{i\pi/4}$z +(1-2i).

   (b) Show that the map of the real axis of the z-plane on to the w-plane by the transformation $W = \frac{1}{z} + i$ is a circle. Find its center and radius. [8+8]

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Set No. 2

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Show that $\int\limits_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1
   (b) Show that β(m,n)= $\int\limits_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$
   (c) Show that $\int\limits_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]

2. (a) Prove that $P_n(0)$=0 for n odd and $P_n\left(0\right) = \frac{(-1)^{\frac{n}{2}} n!}{2^n \left(\frac{n}{2}!\right)^2}$ if n is even.
   (b) Prove that J_2-J_0= 2 J_0” [8+8]

3. (a) Prove that U = $\overset{-x}{e}$ $[(x^2 - y^2)\cos y + 2xy \sin y]$ is harmonic and the analytic function whose real pant is u.
   (b) Separate the real and imaginary parts of Sin h z. [8+8]

4. (a) Evaluate using Cauchy’s integral formula $\int\limits_c \frac{dz}{(z-6z+25)^2}$

      C being the circumference of the ellipse $x^2+4(y-2)^2= 4$
   (b) Evaluate $\int\limits_0^{1+i} (x - y + ix^2) dz$ along
      i. z=0 to 1+I
      ii. The real axis from z=0 to 1 and their along a line parallel to the ‘ imaginary axis from z=1 to 1+i

   [8+8]

5. (a) State and derive Laurent’s series for an analytic function f (z).
   (b) Expand $\frac{1}{(z^2-3z+2)}$ in the region
      i. $0 < | z - 1 | < 1$
      ii. $1 < | z | < 2$. [8+8]

6. (a) Determine the poles of the function and the corresponding residues $\frac{z+1}{z^2(z-2)}$

(b) Evaluate $\int\limits_{c} \frac{Z-3}{Z^2+2Z+5}$ dz where C is the circle using residue theorem

i. $|\ Z\ | = 1$

ii. $|\ Z+1-i\ | = 2$ [6+10]

7. (a) Use method of contour integration to prove that $\int\limits_{0}^{2\pi} \frac{d\theta}{1+a^2-2aCos\theta} = \frac{2\pi}{1-a^2}$,0< a<1

(b) Evaluate $\int\limits_{0}^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

8. (a) Find the image of the region in the z-plane between the lines y=0 and y=Π/2 under the transformation $W = e^z$

(b) Find the image of the line x=4 in z-plane under the transformation $w=z^2$ [8+8]

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1. (a) Show that β(m,n)=2 $\int\limits_{o}^{\pi/2} \sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$ and deduce that $\int\limits_{o}^{\pi/2} \sin^n\theta d\theta = \int\limits_{o}^{\pi/2} \cos^n\theta d\theta = \frac{\Gamma(\frac{1}{2}(n+1)/2))\Gamma\left(\frac{1}{2}\right)}{2\Gamma(\frac{1}{2}(n+2))}$

   (b) Prove that Γ(n) Γ((1-n)= $\frac{\Pi}{\sin n\prod}$

   (c) Show that $\int\limits_{0}^{\infty} x^m e^{-a^2x^2}dx = \frac{1}{2a^{n+1}}\Gamma(n+1)/2$ and hence deduce that $\int\limits_{o}^{\infty} \cos(x^2)dx = \int\limits_{o}^{\infty} \sin(x^2)dx = 1/2\sqrt{\pi/2}$ [5+5+6]

2. (a) When n is a positive integer show that J_n(x) $= \frac{1}{\pi}\int\limits_{0}^{\pi} \cos(n\theta - x\sin\theta)d\theta.$

   (b) Show that x^3 = $\frac{2}{5}$P$_3$(x) + 3/5 P$_1$(x). [8+8]

3. (a) Show that the function u=2 log (x^2+ y^2) is harmonic and find its harmonic conjugate

   (b) Separate the real and imaginary parts of tan hz [8+8]

4. (a) Evaluate using Cauchy's integral formula $\int\limits_{c} \frac{(z+1)dz}{z^2+2z+4}$ where c :$|\,z+1+i\,|\; = 2$

   (b) Evaluate using Cauchy's integral formula $\int\limits_{C} \bar{z}\, from\, z\; = 0\, to\, 4+2i$ along the curve C given by
   i) z=t^2+it
   ii) Along the line z=0 to z=2: and there from z=2: to 4+2i [8+8]

5. (a) State and prove Taylor's theorem.

   (b) Obtain Taylor series to represent the function $\frac{z^2-1}{(z+2)\,(z+3)}$ in the region $|z|\, <\, 2$ [8+8]

6. (a) Find the poles, of f(z) and the residues of the poles which lie on imaginary axis if f(z) $= \frac{(z^2+2z)}{(z+1)^2(z^2+4)}$

(b) Evaluate $\int\limits_C \frac{e^{2z}}{(z+1)^3}$ using residue theorem. [8+8]

7. (a) Show that $\int\limits_0^{2\pi} \frac{d\theta}{a+bSin\theta} = \int\limits_0^{2\pi} \frac{d\theta}{a+bSin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, a > b> 0 using residue theorem.

   (b) Evaluate by contour integration $\int\limits_0^{\infty} \frac{dx}{1+x^2}$ [8+8]

8. (a) Under the transformation w=1/z, find the image of the circle |z-2i|=2.

   (b) Under the transformation $w = \frac{z-i}{1-iz}$, find the image of the circle |z|=1 in the w-plane. [8+8]

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1. (a) Show that $\Gamma(\text{n}) = \int\limits_0^1 (\log 1/y)^{n-1}\, dy$
   (b) Prove that $\int\limits_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}}{n}\frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n}+\frac{1}{2})}$
   (c) $\int\limits_0^1 \frac{x^2 dx}{\sqrt{1-x^4}}$ [5+5+6]

2. (a) Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0(\text{x}) + P_1(\text{x})\ \text{t} + P_2(\text{x})\ \text{t}^2 + ...$
   (b) Write $\text{J}_{5/2}(\text{x})$ in finite form. [8+8]

3. (a) Test for analyticity at the origin for $\text{f(z)} = \frac{x^3 y(y-ix)}{x^6+y^2}$ for $\text{z} \neq 0$
   $= 0$ for $\text{z} = 0$.
   (b) Find all values of z which satisfy (i) $\text{e}^z = 1+\text{i}$ (ii) sinz =2. [8+8]

4. (a) Evaluate $\int\limits_c (y^2 + 2xy)dx + (x^2 - 2xy)dy$ where C is the boundary of the region by y=x^2 and x=y^2
   (b) Evaluate using Cauchy's theorem $\int\limits_c \frac{z^3\, e^{-z}\ dz}{(z-1)^3}$ where c is $|\, z - 1\,| = 1/2$ using Cauchy's integral formula
   (c) Evaluate $\int\limits_{(0,0)}^{(1,1)} (3x^2 + 5y + i(x^2 - y^2)dz$ along y^2=x. [6+5+5]

5. (a) Find the Laurent expansion of $\frac{1}{z^2-4z+3}$, for $1 < |\,\text{z}\,| < 3$.
   (b) Expand the Laurent series of $\frac{z^2-1}{(z+2)(z+3)}$, for $|\,\text{z}\,| > 3$. [8+8]

6. (a) Find the poles and residues at each pole Tanhz
   (b) Evaluate $\int\limits_C \frac{z^3\ dz}{(3-1)^2(z-3)}$ where c is $|\,z\,| = 2$ by residue theorem. [8+8]

7. (a) Evaluate $\int\limits_0^{2\pi} \frac{\sin^2\theta\, d\theta}{a^+ b\cos\theta}$ using residue theorem.

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)\,(x^2+4)}$ using residue theorem. [8+8]

8. (a) Show that the transformation w=z+1/z converts the straight line arg z=a($|a| < \pi/2$) in to a branch of the hyperabola of eccentricity sec a

   (b) Find the bilinear transformation which maps the points (0, 1, ∞) into the points (-1, -2, -i). [8+8]

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