Code No: NR220202 **NR**

II B.Tech II Semester Supplementary Examinations, Apr/May 2006
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Mechatronics, Electronics & Telematics, Metallurgy & Material Technology and Aeronautical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Evaluate $\int_0^1 x^3\sqrt{1-x}dx$ using $\beta-\Gamma$ functions.
   (b) Prove that $\beta(m+1,n)=\frac{m}{m+n}\beta(m,n)$
   (c) Write $\int_0^\infty x^{2n-1}e^{-ax^2}dx$ interms of Γ functions. [5+5+6]

2. (a) Prove that $\int_{-1}^{1}(x^2-1)\,P_{n+1}P_n^1\,dx = \frac{2n(n+1)}{(2n+1)(2n+3))}$
   (b) Prove that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$ [8+8]

3. (a) If f(z) = u+iv is analytic function and u – v = e^x (cos y – sin y), find f(z) in terms of z.
   (b) Find the square root of the complex number 5 + 12i [10+6]

4. (a) Evaluate $\int_c \frac{z^2-2z-2\quad dz}{(z^2+1)^2z}$ where c is $|z-i| = 1/2$ using Cauchy's integral formula
   (b) Evaluate $\int_{(0,0)}^{(1,1)}(3x^2+4xy+ix^2)dz$ along y=x^2
   (c) Evaluate $\int_c \frac{e^{2z}\,dz}{(z^2+\Pi^2)^3}$ where c is $|z| = 4$ using Cauchy's integral formula [5+5+6]

5. (a) Expand as a Taylor series in f(z) $=\frac{2z^3+1}{z^2+1}$ about z=1
   (b) Express f(z) = $\frac{z}{(z-1)(z-3)}$ in a series of positive and negative powers Of (z-1) [8+8]

6. (a) State and prove Residue theorem.
   (b) Evaluate $\int_C \tan z dz$ where C is the circle |z| = 2 using residue theorem
   (c) Evaluate $\int_C \frac{3\cos z}{(2i+3z)}$where C is the unit circle using residue theorem [5+6+5]

7. (a) Evaluate $\int_{-\infty}^{\infty}\frac{\cos x dx}{(x^2+16)(x^2+9)}$ using residue theorem.

(b) If a>e, use Rouche's theorem to prove that the equation $e^z = a\ z^n$ has n roots inside the circle $|z| = 1$ [8+8]

8. (a) Find the image of the domain between x= $\pi/6$ and x=$\pi/3$ under the transformation w=cosz

(b) Find the bilinear transformation which maps the points z=6,i,3 on to the points 0,1,∞. [8+8]

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