Code No: RR322204

Set No. 1

III B.Tech II Semester Regular Examinations, Apr/May 2006
DIGITAL AND OPTIMAL CONTROL SYSTEMS
(Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. Determine the convolution of $f(k) = \left(\frac{1}{4}\right)^k$ and $g(k) = \left(\frac{1}{3}\right)^k$ for all $K \geq 0$. Check your results using Z-transformation. [16]

2. A regulator system has a plant, described by
$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)\,(s+2)\,(s+3)}$
Obtain the discrete-time state variable model. [16]

3. Define controllability and observability of discrete time systems. For the following system,

$$\frac{Y(z)}{U(z)} = \frac{z^{-1}\left(1 + 0.8z^{-1}\right)}{1 + 1.3z^{-1} + 0.4z^{-2}}$$

Determine whether the system is observable and controllable.
[16]

4. (a) What are the two basic transformations used to convert an analog system transfer function to a digital system transfer function? Explain each procedure.
   (b) Explain the design of digital controllers through bilinear transformation.[8+8]

5. A regulator system has the plant characterized by
X(k+1) = AX(k)+Bu(k)
Y(k) = CX(k)
with $A = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$
Compute K so that the control law u(k) = -KX places the closed loop poles at -2+j3.464, -2-j3.464, -5. Give the state variable model of the closed loop system.
[16]

6. (a) Explain the steps involved in solving an optimal control problem.
[4]
   (b) Find the trajectories in the (t, x) plane which will extremize
   $J(\mathrm{x}) = \int_0^{t_1} (t\,\dot{x} + \dot{x}^2)dt$
   in each of the following two cases:

i. t_1=1, x(0) = 1, x(1)=5 [6]

ii. t_1=1, x(0) = 1, x(1) is free. [6]

7. (a) With suitable diagrams illustrate the fixed end-points problem and derive the necessary conditions of variational calculus.

   (b) Find the optimal control u*(t) for the system $\dot{x} = u$; x(0)=1 which minimizes

   $J = \frac{1}{2}x^2(4) + \frac{1}{2}\int\limits_0^4 u^2 dt$ [8+8]

8. (a) Derive the relations required for obtaining an observable realization algorithm of a given transfer matrix T(s).

   (b) Obtain state space controllable realization of a system with transfer matrix.

   $T(s) = \begin{bmatrix} 2(s-1) & s+1 \\ 4 & -s \end{bmatrix} \begin{bmatrix} s+4 & 2(s+1) \\ 0 & s^2-s+4 \end{bmatrix}^{-1}$ [8+8]

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1. Obtain the inverse Z-transform of
   (a) $F_1(z) = \frac{z^{-1}}{(1-z^{-1})(1+1.3z^{-1}+0.4z^{-2})}$ [6]
   (b) $F_2(z) = \frac{1+6z^{-2}+z^{-3}}{(1-z^{-1})(1-0.2z^{-1})}$ [5]
   (c) $F_3(z) = \frac{z^{-1}\left(0.5-z^{-1}\right)}{(1-0.5z^{-1})(1-0.8z^{-1})^2}$ in the closed form. [5]

2. A regulator system has a plant, described by
   $$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)\,(s+2)\,(s+3)}$$
   Obtain the discrete-time state variable model. [16]

3. Consider the following continuous control system
   $$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
   y(t)=x_1(t)
   The control signal u(k) is now generated by processing the signal u(t) through a sampler and zero order hold. Study the controllability and observability properties of the system under this condition. Determine the values of the sampling period for which the discretised system may exhibit hidden oscillation. [16]

4. The figure1 shown below illustrates a typical sampled data system. The transfer functions G_p(s) = 1/s of the controlled plant. Design the data processing unit D(z) which operates as the sampled error signal e_S(t). [16]

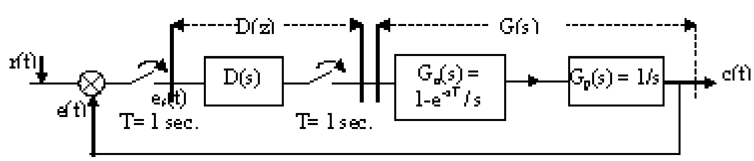

Figure 1:

5. Consider the digital control system X[(k+1)T] =AX(kT)+Bu(kT)
   where $A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
   The state feedback control is described by u(kT) = -KX(kT) where K = [K_1 K_2].

Find the values K_1 and K_2 so that the roots of the characteristic equation of the closed loop system are at 0.5 and 0.7.

[16]

6. (a) State and explain the state regulator problem. Define its performance measure.

   (b) Find the extremals for the functional
   $J(x) = \int_1^{t_1} (2x + \frac{1}{2}\dot{x}^2)dt$; $x(1) = 2$; $x(t_1) = 2$, $t_1 > 1$ is free. [8+8]

7. (a) With suitable diagrams illustrate the one point is fixed end, terminal time t_1 is specified and $x(t_1)$ free end problem and derive the necessary conditions of variational calculus.

   (b) Find the optimal control u*(t) for the system $\dot{x} = u$; $x(0) = 1$ which minimizes
   $J = \frac{1}{2}x^2(4) + \frac{1}{2}\int_0^4 u^2 dt$ [8+8]

8. (a) With step-by-step procedure explain the observable realization algorithm.

   (b) The transfer matrix of a system is

   $$T(s) = R(s)P^{-1}(s) = \begin{bmatrix} s+1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s^2 & 0 \\ -1 & s-1 \end{bmatrix}^{-1}$$

   Obtain controllable realization in the state space form. Is the realization expected to be minimal? [8+8]

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Set No. 3

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. Given the discrete-time system
   y(k) $+a_1$y(k-1) + a_2y(k-2) = b_1u(k) - b_2u(k-1)
   Obtain block diagrams using
   (a) standard programming and [8]
   (b) ladder programming using pure delay elements z^1. [8]

2. (a) Obtain a state space presentation of the following system:
   $$\frac{Y(z)}{U(z)} = \frac{z^{-1}+2z^{-2}}{1+0.7z^{-1}+0.12z^{-2}}$$ [9]
   Choose state variation state matrix as a diagonal matrix.
   (b) State and prove the properties of the state transition matrix of discrete time system [7]

3. Explain Liapunov stability criterion for the linear time variant systems. [16]

4. What is a dead beat response? Explain the characteristic of the poles of the transfer function of a system that has a dead beat response. [16]

5. (a) Enumerate the design steps for pole placement.
   (b) Prove Ackermann's formula for the determination of the state feedback gain matrix K. [8+8]

6. (a) State and explain the minimum - time problems. Describe its performance index. [6]
   (b) Let f(X) = -x_1x_2 and let g(X) $=x_1^2 + x_2^2 - 1$. What are the potential candidates for minima of f subject to the constraint g=0?. Show that the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$actually provide the minima. [10]

7. (a) With suitable diagrams illustrate the one point is fixed end, terminal time t_1 is specified and x(t_1) free end problem and derive the necessary conditions of variational calculus.
   (b) Find the optimal control u*(t) for the system $\dot{x} = u$; x (0) =1 which minimizes
   $$J = \frac{1}{2}x^2(4) + \frac{1}{2}\int_0^4 u^2 dt$$ [8+8]

8. (a) Derive the relations required for obtaining an observable realization algorithm of a given transfer matrix T(s).

(b) Obtain state space controllable realization of a system with transfer matrix.

$$T(s) = \begin{bmatrix} 2(s-1) & s+1 \\ 4 & -s \end{bmatrix} \begin{bmatrix} s+4 & 2(s+1) \\ 0 & s^2-s+4 \end{bmatrix}^{-1}$$ [8+8]

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DIGITAL AND OPTIMAL CONTROL SYSTEMS
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. A slowly changing continuous-time signal f(t) is sampled every T seconds.
   (a) Assuming that the changes in f(t) are very slow compared to the sampling frequency, show that in the z-domain,
   $\frac{1-z^{-1}}{T}$
   corresponds to differentiation. Obtain a block diagram of this differentiator using a pure delay element z^{-1}. [8]
   (b) What is an equivalent integrator in the z-domain? Draw a graph of the output in each case when the input is a unit step sequence. [8]

2. Explain the procedure for discretization of continuous time state space equations. [16]

3. Consider the following continuous control system
   $$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
   y(t)=x_1(t)
   The control signal u(k) is now generated by processing the signal u(t) through a sampler and zero order hold. Study the controllability and observability properties of the system under this condition. Determine the values of the sampling period for which the discretised system may exhibit hidden oscillation. [16]

4. What is a dead beat response? Explain the characteristic of the poles of the transfer function of a system that has a dead beat response. [16]

5. A regulator system has the plant characterized by
   X(k+1) = AX(k)+Bu(k)
   Y(k) = CX(k)
   $$\text{with } A = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$
   Compute K so that the control law u(k) = -KX places the closed loop poles at -2+j3.464, -2-j3.464, -5. Give the state variable model of the closed loop system. [16]

6. (a) State and explain the minimum-energy problems. Describe its performance index.

(b) Show that the extremal for the functional

$J(\mathrm{x}) = \int_0^{\pi/2} (\dot{x}^2 - x^2)dt$

which satisfies the boundary conditions x(0) =0; x(π/2)=1 , is x*(t) =sin t. [8+8]

7. (a) State and explain the Pontryagin's minimum principle.

   (b) Find the points in the three-dimensional euclidean space that minimizes the function

   $f(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) = \mathrm{x}_1^2 + \mathrm{x}_2^2 + \mathrm{x}_3^2$

   and lie on the intersection of the surfaces

   $\mathrm{x}_3 = \mathrm{x}_1\mathrm{x}_2 + 5$

   $\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 = 1$ [8+8]

8. (a) Derive the relations required for obtaining an observable realization algorithm of a given transfer matrix T(s).

   (b) Obtain state space controllable realization of a system with transfer matrix.

   $T(s) = \begin{bmatrix} 2(s-1) & s+1 \\ 4 & -s \end{bmatrix} \begin{bmatrix} s+4 & 2(s+1) \\ 0 & s^2 - s + 4 \end{bmatrix}^{-1}$ [8+8]

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