Code No: RR210501 **Set No.1**

II B.Tech. I Semester Supplementary Examinations, May -2005
DISCRETE STRUCTURES AND GRAPH THEORY
(Common to Computer Science & Engineering, Information Technology, Computer Science & Systems Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Describe the Tautological implications
   (b) Show the following implications without constructing the truth table.
   $P \to Q \Rightarrow P \to (P \wedge Q)$
2. (a) Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. Define the following relation R on A such that (a, b) R (a', b') if and only if a b' = a'b.
   (b) Show that R is an equivalence relation.
   (c) Compute A/R.
3. (a) Define the term 'lattice', clearly stating the axioms.
   (b) Let C be a collection of sets which are closed under intersection and union. Verify whether $(C, \cap, \cup)$ is a lattice.
4. Prove that any 2 simple connected graphs with n vertices, all of degree 2, are isomorphic.
5. Suppose that $(X,\overline{X})$ is an S-D cut in a transport net work (G, K). Prove that $(X,\overline{X})$ and $(\overline{X}, X)$ contain an equal number of edges in common with any directed circuit in G.
6. Prove whether it is always, never, or some times prove that the order in which the nodes are added to the minimum spanning tree by the dijkstra-prim algorithm is the same as the order in which they are encountered in a depth-first traversal.
7. (a) How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where each $x_i \geq 2$?
   (b) Find the number of district triples (x_1, x_2, x_3) of nonnegative integers satisfying the inequality $x_1 + x_2 + x_3 < 6$.
8. Solve the recurrence relation
   S(k) - 0.25 S(k-1) = 0, S(o) = 6.

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Code No: RR210501 **Set No.2**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Explain the predicates with suitable examples.
   (b) Describe the predicate logic.
2. (a) Let A be a set with cardinality n and let R be a relation on A. Then prove that the transitive closure R^α is given by R^α = R U R^2 U - - - - - - - - U R^n
   (b) Let A = $\{1, 2, 3, 4, 5\}$ and R =$\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)\}$ and S = $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 4), (4, 5), (5, 5)\}$. Find the smallest-equivalence relation containing R and S and compute the partition of A that it produces
3. (a) Let L be lattice. Then prove that a $\wedge$ b = a if and only if a $\vee$ b = b.
   (b) Define the dual of a statement in a lattice L.Why does the principle apply to L?
4. (a) Prove that the sequence 5,5,3,3,2,2 is graphic. Draw the graph
   (b) Show that 5,5,3,3,2,2,2 form a graphical sequence
5. (a) The edges of a K$_6$ (Complete Graph on 6vertices) are to be painted either red or blue. Show that for any arbitrary way of painting the edges, there is either a red K$_3$ (a K$_3$ with all its edges painted red) or a blue K$_3$.
   (b) Explain how chromatic number concept can be applied to solve the 'Scheduling Problems'.
6. (a) Write a detailed algorithm for depth-first traversal using an adjacency matrix that just prints the node label as the visit operation. You should trace it using the graphs.
   (b) Prove that each edge in a connected graph will be part of the depth-first traversal tree or will be an edge pointing to a predecessor in the tree.
7. (a) Suppose that JNTU has a residence hall that has 5 single rooms, 5 double rooms, and 3 rooms for 3 students each. In how many ways can 24 students be assigned to the 13 rooms ?
   (b) How many ways are there to distribute 10 balls into 6 boxes with at most 4 balls in the first 2 boxes (that is, if xi = the number of balls in box I, then x$_1$ + x_2 $\leq$ = 4)if
      i. the balls are indistinguishable ?

ii. The balls are distinguishable ?

8. (a) Find a recurrence relation for the number of ways to arrange flags on a flag pole n feet tall using 4 types of flags. Red flags 2 feet high, (or) White, blue and yellow flags each 1 foot high.

   (b) Find a recurrence relation for the number of ways to make a pile of n chips using garnet, gold, red, white and blue chips such that no two gold chips are together.

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Code No: RR210501 **Set No.3**

II B.Tech. I Semester Supplementary Examinations, May -2005
DISCRETE STRUCTURES AND GRAPH THEORY
(Common to Computer Science & Engineering, Information Technology, Computer Science & Systems Engineering and Electronics & Computer Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. Obtain the principal disjunctive and conjunctive normal forms of the following.
   (a) $(\urcorner P V \urcorner Q) \rightarrow (P \rightarrow \urcorner Q)$
   (b) $(Q \rightarrow P)\ (\urcorner P \wedge Q)$
   (c) $PV\ (\urcorner P \rightarrow (QV\ (\urcorner Q \rightarrow R)))$
2. (a) Given S = {1,2,3,....,10}and a relation R on S where R = {<x,y> / x+y + 10}, what are the properties on the relation R?
   (b) Show that if f <x, y> defines the remainder upon the division of y by x, then it is a primitive recursive function.
3. (a) Show that the function $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$ are inverse of each other
   (b) Show that $\{ < x, x > \ /x \in \ N\}$ which defines the relation of equality is primitive recursive.
4. (a) Define 1 - and 2- isomorphism with one example each.
   (b) If G_1 and G_2 are two 1-isomorphic graphs then the rank of G_1 is equal to the rank of G_2 and the nullity of G_1 is equal to the nullity of G_2 .
5. (a) Show that a simple graph with n vertices ($n \geq 3$) in which each vertex has degree at least n/2 has a Hamiltonian cycle.
   (b) How many different Hamiltonian cycles are there in Kn (a complegraph on n vertices)?
6. Prove whether it is always, never, or some times prove that the order in which the nodes are added to the minimum spanning tree by the dijkstra-prim algorithm is the same as the order in which they are encountered in a breadth-first traversal.
7. (a) Explain the terms
      i. Disjunctive counting and
      ii. Sequential counting.
   (b) How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8, and 9 if no repetitions are allowed?

8. Solve the recurrence relation
   S(k) - 10 S(k-1) + 9S(k-2) = 0, S(o) = 3, S(1) = 11.

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Code No: RR210501 **Set No.4**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Write the following statement in symbolic form:
   "The crop will be destroyed if there is a flood"
   (b) Construct the truth table of the following formula.
   $\urcorner(PV(Q \wedge R)) \rightleftarrows ((PVQ) \wedge (PVR)$
2. (a) List all the permutations on A = {a,b,c}
   (b) Let X= {1,2,3, .. ,25} and R= { (x,y) / x-y is divisible by 5 } be a relation on X. Show that R is an equivalence relation.
3. (a) Let L be a lattice. Then prove that the relation $a \leq b$ defined by either $a \wedge b = a$ or $a \vee b = b$ is a partial ordering on L.
   (b) Let X = { 1, 2, 3, 4 }. Define a function f : X→X such that $f \neq I_x$ and is one to one. Find f^2, f^3, f^{-1}, $f \bullet f^{-1}$. Can you find another one to one function g : X→X such that $g \neq I_x$ but $g \bullet g = I_x$.
4. (a) Find the rank and nullity of the complete graph K_n
   (b) Prove that a connected graph G remains connected after removing an edge e from G if and only if e belongs to some circuit in G.
5. (a) Which of the following non planar graphs have the property that the removal of any vertex and all edges incident with that vertex produces a planar graph?
   i. K_5.
   ii. K_6.
   iii. $K_{3,3}$.
   iv. $K_{3,4}$.
   (b) Let n be a positive integer. Show that a sub graph induced by a non empty subset of the vertex set of K_n is a complete graph.
6. (a) Implement a graph so that the lists of header nodes and arc nodes are circular.
   (b) Implement a graph using linked lists so that each header node heads two lists. One containing the arcs emanating from the graph node and the other containing the arcs terminating at the graph node.
7. (a) How many ways can 20 similar books be placed on 5 different shelves ?

(b) Enumerate the number of ways of placing 20 indistinguishable balls into 5 boxes where each box is nonempty.

8. Solve the recurrence relation
S(k) - 0.25 S(k-1) = 0, S(o) = 6.

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