Code No: RR221101 **Set No.1**

II B.Tech. II Semester Regular Examinations, April/May -2005
PROBABILITY THEORY & STOCHASTIC PROCESS
(Bio-Medical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Define the following and give one example for each.
   i. Sample space.
   ii. Event.
   iii. Mutually exclusive events
   iv. Collectively exhaustive events.

   (b) In three boxes, there are Capacitors as shown in the following table:

Value in mf	Number in box		
↓	→		
	1	2	3
1-0	70	80	145
0.1	55	35	75
0.01	20	95	25

   An experiment consists of first randomly selecting a box (assume that each box has the same probability of selection) and then randomly selecting a capacitor from the chosen box.
   i. What is the probability of selecting 0.01uf capacitor, given that the box2 is chosen?
   ii. If a 0.01 mf capacitor is chosen, what is the probability that it came from the second box?

2. (a) Define Conditional Probability mass function.

   (b) If two random variables have the joint probability density
   $f(x_1, x_2) = \frac{2}{3}(x_1 + 2x_2)\ for\, 0 < x1 < 1, 0 < x2 < 1$

   $= 0$ elsewhere. Find
   i. the marginal density of x_2
   ii. Conditional density of the first given that the second takes on the value x_2.

   (c) A pair of dice is tossed. Define a random variable X to be the difference of the face values turned up. Determine the probability mass function of X.

3. (a) State and prove any four properties of mean of a random variable X.

(b) Prove that the density function of sum of two statistically independent random variables is the convolution of their individual density functions.

4. Let the Random process be given as = Z(t) = x(t) cos $[\omega_0 t + \theta\ ]$ where x(t) in stationary Random process with E[x(t)]=0 and $E[x^2(t)] = \sigma_x^2$

   (a) If $\theta = 0$ find E[Z(t)] and $E[Z^2]$ if Z(t) stationary.

   (b) If θ is a random variable independent of x(t) and uniformly distributed over the interval $(-\Pi, \Pi)$) show that E[Z(t)] = 0 and $E[Z^2(t)] = \frac{\sigma_x^2}{2}$.

5. White noise n(t) with G(f) = $\eta/2$ is passed through a low pass RC network with a 3dB frequency f_c.

   (a) Find the autocorrelation $R(\tau)$ of the out put noise of the network.

   (b) Sketch P$R(\tau) = R(\tau)/R(0)$

   (c) Find ϖc(t) such that P$(\tau)\pounds \leq 0.1$.

6. (a) What do you understand by noise power spectral density?

   (b) How the autocorrelation function of the White noise represented? What is its significance?

7. (a) Show that the effective noise temperature of 'n' networks in cascade is given by,$T_e = T_{e1} + T_{e2}/_{g1} + T_{e3}/g_1g_2 + \ldots\ldots\ldots\ldots + T_{en}/g_1g_2\ldots\ldots g_{n-1}$)

   (b) A low noise receiver for satellite ground station consists of the following stages
   Antenna with $T_i = 125^K$
   Waveguide with a loss of 0.5dB
   Power amplifier with g_a = 30dB, $T_e = 6^0K$, $B_N = 20MHz$
   TWT amplifier with g_a = 16dB, F= 6dB, B_N = 20 MHz
   Calculate the effective noise temperature of the system.

8. (a) Consider an AWGN channel with $\frac{S}{N} = 10^4$. Find the maximum rate for reliable information transmission when, B = 1 KHz, 10 KHz and 100 KHz.

   (b) The Binary Erasure Channel (BEC) has two source symbols 0 and 1, and three destination symbols 0, 1 and E, where E denotes a detected but uncorrectable error. The forward transition probabilities are,

$P(\frac{0}{0}) = 1-\alpha$	$P(\frac{E}{0}) = \alpha$	$P(\frac{1}{0}) = 0$
$P(\frac{0}{1}) = 0$	$P(\frac{E}{1}) = \alpha$	$P(\frac{1}{1}) = 1-\alpha$

   I (x, y) is maximum when source symbols are equiprobable. Find C_s (channel capacity) in terms of α.

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1. (a) Explain the concept of random variable.
   (b) What is the probability of picking an ace and a king from a deck of 52 cards?
   (c) A box contains 4-point contact diodes and 6 alloy junction diodes. What is the Probability that 3 diodes picked at random contain at least two point contact Diodes?

2. (a) Define joint distribution and Joint probability density function for the two random variables X and Y.
   (b) Let X and Y be jointly continuous random variables with joint density function
   $f(x,y) = xy\exp\left[-\frac{1}{2}(x2+y2)\right]; x>0, y>0$
   $=0$ otherwise
   Check whether X and Y are independent. Find
   i. $P(X \le 1, Y \le 1)$and
   ii. $P(X+Y \le 1)$

3. (a) Find the moment generating function of the random variable having probability density function $fx(x) = x, 0 \le x \le 1$
   $= 2-x, 1 \le x \le 2$
   $= 0$, else where
   (b) Find the moment generating function of the random variable whose moments are $m_r = (r+1)!2^r$

4. (a) State and prove properties of cross correlation function.
   (b) Consider the Random process x(t)= A $\cos(\omega_0 t+\theta)$ where A and ω_0 are real constants and θ is a random variable uniformly distributed on the interval $\left(0, \frac{\Pi}{2}\right)$ find the average power $P_x x$ in x(t).

5. (a) Derive the relation between PSDs of input and output random process of an LTI system.
   (b) X(t) is a stationary random process with zero mean and auto correlation $R_{XX}(\tau)e^{-2|\tau|}$ is applied to a system of function $H(w) = \frac{1}{jw+2}$. Find mean and PSD of its output.

6. (a) What are the sources of flicker noise and how can it be reduced?
   (b) How noise equivalent bandwidth of a electronic circuit can be estimated?

7. (a) An amplifier has input and output impedances of 75 ohm, 60dB power gain, and a noise equivalent bandwidth of 15KHz. When a 75 Ω resistor at 290^0K is connected to the input, the output rms noise voltage is 75microvolt. Determine the effective noise temperature of the amplifier assuming that the meter is impedance matched to the amplifier.

   (b) List the devices in which narrowband noise can be present.

8. (a) A source is transmitting six messages with probabilities 0.3, 0.25, 0.15, 0.12, 0.10 and 0.08 respectively. Find the binary "HUFFMAN" Source code for the above messages and code efficiency.

   (b) Consider an AWGN Channel with 4 MHz bandwidth and noise PSD is $\frac{\eta}{2}= 10^{-12}\frac{\text{w}}{\text{Hz}}$.. The signal power is required at the receiver is 0.1 mw. Find the capacity of the channel.

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1. (a) Explain why there must be a mistake in each of the following statements:
   i. If the probability that an ore contains Uranium is 0.28, the probability that it does not contain Uranium is 0.62.
   ii. A Company is working on the construction of two shopping centers. The probability that the larger of the two shopping centers will be completed is 0.35 and the probability that both shopping centers will be completed on time is 0.42.
   iii. The probability that a student gets A in a particular course is 0.32 and the probability that he will get either an A or a B is 0.27.

   (b) A jar contains 52 badges numbered 1 to 52.Suppose that the numbers 1 thro 13 are considered lucky. A sample of size 2 is drawn from the jar with replacement. What is the probability that
   i. both badges drawn will be lucky?
   ii. Neither badge will be lucky?
   iii. Exactly one of the badges drawn will be lucky.
   iv. at least one of the badges will be lucky.

2. (a) Explain the Rayleigh probability density function.

   (b) Find the mean value, the mean squared value and the cumulative distribution functio for the Rayleigh distribution with parameter $\alpha > 0$, , specified by the pdf $f(x) = \frac{x}{\alpha^2} \exp\left\{-\frac{1}{2}\frac{x}{\alpha}\right\}$

3. (a) Find the density function whose characteristic function is $\exp(-|t|)$.

   (b) Let X be a continuous random variable with pdf fx (x)=8/ x^3, x > 2. Find E[W] where W = X/3

4. (a) Which are the following are suitable auto correlation functions?
   i. $A\cos\omega_0\tau$
   ii. $A\Pi(\frac{\iota}{\tau_0})$where$\Pi$(x) is a unit area rectangular function

   (b) Suppose we are given a cross power spectrum defined by
   S_{xx} (ω) = a+(jb ω /w) ; -W < ω < W
   = 0 : Elsewhere
   Where W > 0, a and b are real constants. Find the cross correlation function.

5. (a) Find the PSD of a random process where z(t)= X(t) + y(t) where x(t) and y(t) are zero mean, individual random process.

(b) A wss random process x(t) is applied to the input of an LTI system whose impulse response is $5t.e^{-}2t$. The mean of x(t) is 3. Find the output of the system.

6. (a) Explain how the available noise power in an electronic circuit can be estimated.

   (b) What are the different noise sources that may be present in an electron devices?

7. (a) What are the precautions to be taken in cascading stages of a network in the point of view of noise reduction?

   (b) What is the need for band limiting the signal towards the direction increasing SNR.

8. (a) For a system the bandwidth is 4 KHz and $\frac{S}{N}$ ratio is 14. If the bandwidth is increased to 5 KHz, find the required $\frac{S}{N}$ ratio to have the same channel capacity and find the percentage change in signal power.

   (b) Using the definition of H (x) and H (X/y), Show that, $I(x, y) = \sum_{x,y} P(x, y) \log \frac{P(x,y)}{P(x),P(y)}$

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1. (a) Define Probability density function and obtain the relationship between probability and probability density.
   (b) Consider the probability density $f(x) = a\,e^{-b|x|}$ where x is a random variable Whose allowable values range from x = $-\infty$to∞. Find
      i. the CDF F (x)
      ii. the relationship between a and b. and
      iii. the probability that the outcome x lies between 1 and 2.

2. The Rayleigh density function is given by $f(x) = xe^{-x^2/2}\, x \geq 0$
   $= 0$ x < 0
   (a) Prove that f (x) satisfies the properties of the p.d.f.
      i. f(x)$\geq$ for all x and
      ii. $\int_{\infty}^{\infty} f(x)\,dx = 1$
   (b) Find the distribution function F (x)
   (c) Find $P(0.5 < x \leq 2)$
   (d) Find P(0.5 $\leq$ x $<$ 2).

3. (a) Prove that moment generating function of the sum of two independent variables is the product of their moment generating function.
   (b) Let X and Y be independent random Variables, prove that Var (XY) = Var (X) Var (Y) if E[X] = E[Y] = 0

4. (a) Distinguish between stationary and non stationary Random process.
   (b) Consider a train of Rectangular pulses having as amplitude of 2 volts and widths which are either $1\mu s$ or $2\mu s$ with equal probability. The meantime between pulses is 5 μs . Find the PSD Gn(f) of the pulse train.

5. White noise n(t) with PSD= $\frac{n}{2}$ is passed through a low pass RC network with a 3 db frequency f_c.
   (a) Find the auto correlation $R(\tau)$) of the o/p noise of the network.
   (b) Sketch $\rho(t) = \frac{R(\tau)}{R(o)}$

6. Give reasons for the following:

(a) In any communication system the first stage must have low noise operation.

(b) Describe how FET gives low noise performance compared to BJT.

7. (a) Derive the equation for narrow band noise and illustrate all its properties

(b) Show their noise figure F of a n/w is given by F= $\frac{Go(f)}{K^2Gin(f)}$ whereGo(f), Gin(f), and K are respectively open circuited voltage, spectral density and the voltage gain of n/w.

8. (a) A source is transmitting two symbols A and B with P (A) =$\frac{1}{16}$ and P (B) = $\frac{15}{16}$ Construct a source code to provide a code efficiency of around 50%.

(b) A binary data source has P (0) = $\frac{3}{8}$, P (1) = $\frac{5}{8}$. Due to noise in the channel, P$\left(\frac{1}{0}\right)$=$\frac{3}{4}$, $P\left(\frac{0}{1}\right) = \frac{1}{16}$. Find the conditional entropy H $\frac{y}{x}$ and maximum entropy of 'x' where 'x' is source and 'y' is receiver.

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