Code No: RR220202 **Set No.1**

II B.Tech. II Semester Regular Examinations, April/May -2005
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\beta(\mathrm{m_1 n}) = \Gamma\,(m)\,\Gamma\,(n)\,/\Gamma\,(m+n)$
   (b) Show that $\int\limits_0^1 \frac{x^n}{\sqrt{1-x^2}}dx = \frac{2.4.6....(n-1)}{1.3.5....n}$ where n is an odd integer.
   (c) Show that $\int\limits_0^{\pi/2} \sqrt{\tan\theta}\quad d\theta = \frac{\Gamma(1/4)\Gamma(3/4)}{2}$
2. (a) Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0$(x) + P_1(x) t + P_2(x) t^2+ ....
   (b) Express $\mathrm{J}_{5/2}$(x) in finite form.
3. (a) Show that f(x,y) = x^3y − xy^3 + xy +x +y can be the imaginary part of an analytic function of z = x+iy.
   (b) Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|Real\ f(z)|^2 = 2|f'(z)|^2$ where w =f(z) is analytic
4. (a) Evaluate using Cauchy's theorem $\int\limits_c \frac{zdz}{(z^2-6z+25)^2}$ where c :$|\,z-(3+4i\,| = 4$.
   (b) Evaluate using Cauchy's integral formula $\int\limits_0^{1+i} z^2\ dz$along y = x^2
   (c) Prove that $\int\limits_C \frac{dz}{(z-a)} = 2\pi i$ where C is given by the equation 1z-a1=r
5. (a) Expand cos h z as a maclaurins series if $|z| < \infty$
   (b) Find the Laurent series expansion of the function $\frac{z^2-1}{(z+2)\,(z+3)}$ if 2 $< |z| <$ 3
6. (a) Determine the poles of the function and the corresponding residues (i) $\frac{z+1}{z^2(z-2)}$
   (b) Evaluate $\oint_c \frac{dz}{Sinhz}$, where C is the circle | z | = 4 using residue theorem.
7. (a) Evaluate $\int\limits_0^{\pi} \frac{(1+2\cos\theta)d\theta}{(5+4\cos\theta)}$ using residue theorem.
   (b) Evaluate $\int\limits_{-\infty}^{\infty} \frac{x^2dx}{(x^2+a^2)^2}$, a>0 using residue theorem
8. (a) Find and plot the image of triangular region with vertices at (0,0), (1,0) (0,1) under the transformation w=(1-i) z+3.

(b) If $w = \frac{1+iz}{1-iz}$ find the image of |z| <1.

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Set No.2

II B.Tech. II Semester Regular Examinations, April/May -2005
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. Evaluate the following using $\beta - \Gamma$ functions.

   (a) $\int\limits_0^{\pi/2} \sin^{9/2}\theta \cos^5 \theta d\theta$

   (b) $\int\limits_0^1 e^{-x^3} x^{11/3} dx$

   (c) $\int\limits_0^1 \frac{x^4 dx}{\sqrt{1-x^2}}$.

2. (a) Show thatt J_{n-1} (x) + J_{n+1}(x) = $\frac{2n}{x} J_n(x)$.

   (b) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

   (c) Show that (n+1) P_{n+1} (x) - (2n+1) x P_n(x) + n P_{n-1} (x) = 0.

3. (a) Show that f(x,y) = $x^3y - xy^3$ + xy +x +y can be the imaginary part of an analytic function of z = x+iy.

   (b) Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |Real\ f(z)|^2 = 2|f'(z)|^2$ where w =f(z) is analytic

4. (a) Evaluate $\int\limits_c \frac{dz}{e^z(z-1)^3}$ where c: | z | = 2 Using Caucy's integral theorem.

   (b) Evaluate $\int\limits_{-2+i}^{5+i} z^3 dz$ using Cauchy's integral formula along y = x

   (c) $\int (x+y)dx + x^2y\, dy$ along y=x^2 from (0,0) to (3,i)

5. (a) Expand f(z) = $\frac{e^{2z}}{(z-1)^3}$ about z=1 as a Laurent series. Also find the region of convergence.

   (b) Find the Taylor series for $\frac{z}{z+2}$ about z=1, also find the region of convergence

6. (a) Determine the poles of the function and the corresponding residues

$$\frac{(2z+1)^2}{(4z^3+z)}$$

(b) Evaluate $\int\limits_{C} \frac{(\sin \pi z^2 + \cos \pi z^2) dz}{(z-1)^2 (z-2)}$ where e is the circle |z| = 3 using residue theorem

7. (a) Use method of contour integration to prove that $\int\limits_{0}^{2\pi} \frac{d\theta}{1+a^2-2aCos\theta} = \frac{2\pi}{1-a^2}$, $0 <$ a< 1

   (b) Evaluate $\int\limits_{0}^{\infty} \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem.

8. (a) show that the function w=4/z transforms the straight line x=a in the z-plane into a circle in the w-plane

   (b) Find the bilinear transformation which maps the points z=∞ , i,0 onto the points w=0,1,∞

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Code No: RR220202 **Set No.3**

II B.Tech. II Semester Regular Examinations, April/May -2005
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Define Gamma function and evaluate $\Gamma(1/2)$
   (b) Show that $\Gamma(1/2)\Gamma(2n) = 2^{2n-1}\,\Gamma n\;\Gamma(n+1/2)$
   (c) Define Beta function and show that β(m,n) = β(n,m)
2. (a) Using Rodrigue's formula prove that $\int\limits_{-1}^{1} x^m P_n(x)dx = 0$ if m < n
   (b) Prove that $J_0(x) = \frac{1}{\pi}\int\limits_0^{\pi}\cos(x\cos\theta)d\theta$.
3. (a) If f(z) is an analytic function, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.
   (b) If tan log (x+iy) = a + i b where $a^2 + b^2 \neq 1$ prove that tan log $(x^2+y^2) = \frac{2a}{1-a^2-b^2}$
4. (a) Evaluate $\int\limits_c \frac{Cos\, z - \sin\, z\;\; dz}{(z+i)^3}$ with c: $|\,z\,| = 2$ using Cauchy's integral formula
   (b) Evaluate $\int\limits_{1-i}^{2+i} 2x+1+iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula
5. (a) State and prove Taylor's theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3< |z+2| <5.
6. (a) Find the poles, of f(z) and the residues of the poles which lie on imaginary axis if f(z) = $\frac{(z^2=2z)}{(z+1)^2(z^2+4)}$
   (b) Evaluate $\int\limits_C \frac{e^{2z}}{(z+1)^3}$ using residue theorem.
7. (a) Evaluate $\int\limits_0^{2\pi}\frac{d\theta}{(a+b\cos\theta)^2}$ a> b>0, using residue theorem.
   (b) Evaluate $\int\limits_0^{\alpha}\frac{dx}{(x^2+1)^3}$

8. (a) Find the image of the infinite strip bounded by x=0 and x=$\pi/4$ undet the transformation w=cosz

   (b) Show that the transformation w=(5-4z)/(4z-2) transform the circle |z|=1 into a circle of radius unity in w- plane and find the centre of the circle

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Code No: RR220202 **Set No.4**

II B.Tech. II Semester Regular Examinations, April/May -2005
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Show that $\int\limits_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1
   (b) Show that β(m,n)= $\int\limits_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$
   (c) Show that $\int\limits_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$
2. (a) Prove that $P_n(0)$=0 for n odd and $P_n\,(0) = \frac{(-1)^{\frac{n}{2}} n!}{2^n \left(\frac{n}{2}!\right)^2}$
   (b) Prove that J_2-J_0= 2 J_0”
3. (a) Derive Cauchy Riemann equation in polar coordinates.
   (b) Prove that the function f(z) $= \bar{z}$ is not analytic at any point.
   (c) Find the general and the principal values of (i) log $_e$ (1+$\sqrt{3}$i) (ii) log $_e$(-1).
4. (a) Evaluate $\int\limits_c \frac{z^3 - \sin 3z \;\; dz}{(z-\frac{\pi}{2})^3}$ with c: $|\,z\,| = 2$ using Cauchy’s integral formula
   (b) Evaluate $\int\limits_C (z^2 + 3z + 2)$where C is the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$ between the points ($\prod$a, 2a)
5. (a) For the function f(z)= $\frac{2z^3+1}{z\,(z+1)}$ find Taylor’s series valid in a neighbourhood of z=I
   (b) Find Laurent’s series for f(z) $= \frac{1}{z^2\,(1-z)}$ and find the region of convergence
6. (a) Find the poles and residues at each pole $\frac{ze^z}{(z-1)^3}$
   (b) Evaluate $\int\limits_C \frac{2e^z dz}{z(z-3)}$ where C is |z| = 2 by residue theorem.
7. (a) Show that $\int\limits_0^\pi \frac{Cos 2\theta}{1-2aCos\theta+a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$, (a^2 < 1) using residue theorem.
   (b) Show by the method of contour integration that $\int\limits_0^\infty \frac{Cosmx}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3}(1+ma)e^{-ma}$, (a $>$ 0 , b $>$ 0).

8. (a) Find the image of the infinite strip $0<y<1/2$ under the transformation $w=1/z$.
   (b) Find the bilinear transformation which maps the points $(-1, 0, 1)$ into the points $(0, i, 3i)$.

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