Code No: NR220202 **NR**

II B.Tech II Semester Supplementary Examinations, April/May 2005
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Mechatronics, Electronics & Telematics, Metallurgy & Material Technology and Aeronautical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Show that $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q-1}dx = 2^{p+q-1}\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.
   (b) Show that $\int_{0}^{1} \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{\beta(m,n)}{a^n(1+a)^m}$

2. (a) Show that $J_n(x)$ satisfies the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0$
   (b) Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n P_n(x)$.

3. (a) Show that $w = z^n$ (n , a positive integer) is analytic and find its derivative.
   (b) If $w = f(z)$ is an analytic function, then prove that the family of curves defined by $u(x,y) =$ constant cuts orthogonally the family of curves $v(x,y) =$ constant.
   (c) If $\alpha + i\beta = \tan h\,(x + i\,\pi/4)$ prove that $\alpha^2 + \beta^2 = 1$

4. (a) Evaluate using Cauchy's Integral Formula $\int_c \frac{(z+1)\,dz}{z^3-4z}$ where c is $|z+2| = 3/2$
   Using Cauchy's Integral Formula evaluate
   (b) Evaluate $\int_C z^3 dz$ where C is the curve $x=t, y=t^2$
   (c) $\int_c \frac{e^{3z}\,dz}{(z+i)^4}$ where c is $|z| = 3$ using Cauchy's integral formula

5. Expand $\frac{1}{z(z^2-3z+2)}$ for the regions
   (a) $0 < |z| < 1$
   (b) $1 < |z| < 2$
   (c) $|z| > 2$

6. (a) State and prove residue theorem.
   (b) Using residue theorem Evaluate $\int(4-3z)/(z^2-z)\,dz$ Where C is the circle $|z|=2$..

7. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2}$ using residue theorem.
   (b) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+25)^2}$ using residue theorem.

8. (a) Find the image of the semi-infinite strip x>0, 0<y<z under the transformation w=iz + 1. show the region graphically.

   (b) In the transformation $z = \frac{1-w}{1+w}$show that the positive half of the w-plane given by u $\geq$ 0 corresponds to the circle |z| $\leq$1 in the z-plane

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