

II B.Tech II Semester Supplementary Examinations, April/May 2005
MATHEMATICS-III

(Common to Electrical & Electronic Engineering, Electronics &
 Communication Engineering, Electronics & Instrumentation Engineering,
 Bio-Medical Engineering, Electronics & Control Engineering, Electronics &
 Telematics and Metallurgy & Material Technology)

Time: 3 hours

Max Marks: 70

Answer any FIVE Questions
All Questions carry equal marks

1. Evaluate the following using $\beta - \Gamma$ functions

(a) $\int_0^1 x^7(1-x)^5 dx$

(b) $\int_0^{\pi/2} \sin^5 \theta \cos^{7/2} \theta d\theta$

(c) $\int_0^{\infty} y^{-1/2}(1-e^{-y})dy$

2. (a) Prove that $1 + \frac{1}{2} P_1(\cos \theta) + \frac{1}{3} P_2(\cos \theta) + \dots = \log \left(\frac{1 + \sin \frac{\theta}{2}}{\sin \frac{\theta}{2}} \right)$

(b) Prove that $\frac{1}{2} x J_n = (n+1) J_{n+1} - (n+3) J_{n+3} + (n+5) J_{n+5} - \frac{x}{2} J_{n+6}$

3. (a) Prove that $U = e^{-x} [(x^2 - y^2) \cos y + 2xy \sin y]$ is harmonic and the analytic function whose real part is u .

(b) Separate the real and imaginary parts of $\sinh z$.

4. (a) $\int_c \frac{(z^2 - 4) dz}{z^2(z+2)^2}$ with $c: |z+2| = 1$ using Cauchy's integral formula

(b) Evaluate $\int_{-1+i}^{2+i} (x^2 - y^2 + i xy)^{dz}$ using Cauchy's integral formula along $y=x^2$.

5. (a) State and prove Taylor's theorem.

(b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$.

6. (a) Find the poles and residues at each pole $\frac{(2z+1)}{(z^2-z-2)}$.

(b) Evaluate $\int_C \frac{3 \sin z \cdot dz}{(z^2 - \frac{\pi^2}{4})}$ where C is $|z| = \Pi$ by residue theorem

7. (a) Evaluate $\int_0^{\pi} \frac{a d\theta}{a^2 + \sin^2 \theta}$ using residue theorem.

(b) Evaluate $\int_0^{\infty} \frac{dx}{(x^4+16)}$ using residue theorem.

8. (a) If $w = z + a^2/z$. prove that when z describes the circles $x^2 + y^2 = a^2$, w describes a straight line and find its length.
- (b) Find the mobius transformation which maps $(1, -i, 2)$ z -plane onto $(0, 2, -i)$ in w -plane.
