

**III B.Tech II Semester Supplementary Examinations, April/May 2005**  
**MATHEMATICS-III**

( Common to Mechanical Engineering and Production Engineering)

Time: 3 hours

Max Marks: 70

Answer any FIVE Questions  
 All Questions carry equal marks

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1. (a) Evaluate

i.  $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$

ii.  $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta$

(b) Prove that  $\iiint_V x^{l-1} y^{m-1} z^{n-1} dx dy dz = \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n+1)}$

2. State and prove the necessary and sufficient conditions for the function  $f(z)$  to be differentiable.3. (a) If  $w = u + iv$  represents the complex potential for an electric field and  $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$ , determine the function  $\phi$ 

(b) If  $f(z)$  is a regular function of  $z$  prove that  $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$

4. (a) Evaluate  $\int_{1-i}^{2+i} (2x + iy - 1) dz$ , along the two paths.

i.  $x = x + 1, y = 2x - 1$

ii. The straight line joining  $1 - i$  and  $2 + i$

(b) State and prove Cauchy's integral formula.

5. Expand  $f(z) = \frac{1}{(z-1)(z-2)}$  in the region.

(a)  $|z| < 1$

(b)  $1 < |z - 1| < 2$

(c)  $|z| > 2$

(d)  $0 < |z - 1| < 1$ .

6. (a) Determine the poles of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  and the residue at each pole.

(b) Evaluate  $\frac{1-2z}{z(z-1)(z-2)} dz, C : |z| = 1.5$ .

7. (a) Evaluate  $\int_C \frac{z \sec z}{(1-z)^2} dz, C : |z| = 3$ 

(b) Show that  $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{1-2a \cos \theta + a^2} = \frac{2\pi a^2}{1-a^2}, (a^2 < 1)$

8. (a) Show that the transformation  $W = \frac{i(1-z)}{(1+z)}$  maps the circle  $|z| = 1$  into the real axis of the w-plane and the interior of the circle  $|z| < 1$  into the upper half of the w-plane.
- (b) Find the bilinear transformation which maps 1, i, -1, to 2, i, -2 respectively. Find the fixed and critical points of the transformation.

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