Code No: RR322204

Set No.1

III B.Tech. II Semester Regular Examinations, April/May -2005
DIGITAL AND OPTIMAL CONTROL SYSTEMS
(Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. The signal $f(t) = 5\sin(20\pi t) + 2\Pi\left(\frac{t-0.14}{0.16}\right)$
   where Π(x) is the unit pulse signal defined by $\Pi(x) = \begin{cases} 1 & if\ |x| \le \frac{1}{2} \\ 0 & otherwise \end{cases}$
   is sampled at 25 samples per second. Determine the Z-transform of f(kT) for $0 \le k \le 8$.

2. Obtain a state space representation of the system shown below figure 1

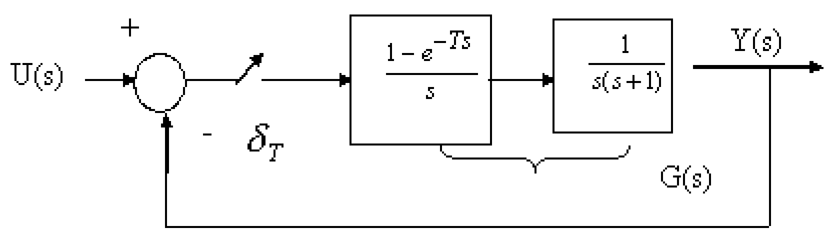

Figure 1:

3. Define controllability and observability of discrete time systems. For the following system,

$$\frac{Y(z)}{U(z)} = \frac{z^{-1}\left(1 + 0.8z^{-1}\right)}{1 + 1.3z^{-1} + 0.4z^{-2}}$$

   Determine whether the system is observable and controllable.

4. Explain the digital implementation of PI controller and lead lag controller in detail with suitable diagrams.

5. (a) Enumerate the design steps for pole placement.
   (b) Prove Ackermann's formula for the determination of the state feedback gain matrix K.

6. (a) State and explain the tracking problem. Define its performance measure.
   (b) The system
   $\dot{x} = -x + u$
   is to be transferred from x(0) = 5 to x(1) = 0 such that
   $J = \int_0^1 (\dot{u})^2 dt$ is minimized. Find the optimal control.

7. (a) Explain the concepts of variational calculus.
   (b) Explain the formulation of Variational CalCulus using Himiltonian method.
8. (a) Explain the input decoupling zeros and output decoupling zeros.
   (b) Obtain minimal differential operator realization of T(s) given below. Further, convert the D.O. form to the state space form.

$$T(s) = \begin{bmatrix} \frac{s+1}{s^2} & \frac{s+2}{s^2+1} \\ \\ \frac{2}{s} & \frac{2s+3}{s^2+1} \end{bmatrix}$$

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1. Obtain the inverse Z-transform of the following in the closed form.
   (a) $F_1(z) = \frac{0.368z^2+0.478z+0.154}{z^2(z-1)}$
   (b) $F_2(z) = \frac{2z^3+z}{(z-1)^2(z-1)}$
   (c) $F_3(z) = \frac{z+2}{z^2(z-2)}$

2. Consider the discrete control system represented by
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1.5 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} \mathrm{u(k)}$$
$$\mathrm{y(k)} = \begin{bmatrix} -3 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{x}_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \end{bmatrix} u(k)$$
where,
$$u(k) = \left(\tfrac{1}{2}\right)^k, k \geq 0$$
Find the response y(k), $K \geq 0$

3. Consider the following continuous control system
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\mathrm{y(t)} = \mathrm{x_1(t)}$$
The control signal u(k) is now generated by processing the signal u(t) through a sampler and zero order hold. Study the controllability and observability properties of the system under this condition. Determine the values of the sampling period for which the discretised system may exhibit hidden oscillation.

4. What are PID controllers? Compare its performance with PI controllers and PD controllers. Explain digital PID controller in detail.

5. Consider the system defined by $\dot{X}$ = Ax + Bu
$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
by using the state feedback control u = -Kx, it is desired to have the closed loop poles at s = -2 ± j 4 and s= -10. Determine the state feedback gain matrix K.

6. (a) Explain the steps involved in solving an optimal control problem.
   (b) Find the trajectories in the (t, x) plane which will extremize
   $J(\mathrm{x}) = \int_0^{t_1} (t\,\dot{x} + \dot{x}^2)dt$
   in each of the following two cases:
      i. t_1=1, x(0) = 1, x(1)=5
      ii. t_1=1, x(0) = 1, x(1) is free.

7. (a) Explain the concepts of variational calculus.
   (b) Explain the formulation of Variational CalCulus using Himiltonian method.

8. Break up the following transfer matrices into R(s) and P(s)
   (a) $T(s) = R(s)P^{-1}(s)$
   (b) R(s) and P(s) are relatively right prime,
   (c) P(s) is column proper
      i. $T(s) = \begin{bmatrix} \frac{s+1}{s^2} & \frac{s+2}{s^2+1} \\ \frac{2}{s} & \frac{2s+3}{s^2+1} \end{bmatrix}$
      ii. $T(s) = \begin{bmatrix} \frac{(s-2)(s+1)}{s(s-1)^2} & \frac{1}{(s-1)^2} \\ -\frac{1}{s} & 0 \\ \frac{2}{s(s-1)} & \frac{1}{s-1} \end{bmatrix}$

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1. Given the transfer functions
   (a) $F_1(s) = \frac{s+1}{(s+2)(s+3)}$
   (b) $F_2(s) = \frac{1-e^{4s}}{s} \qquad \frac{s+1}{(s+2)^2}$

   Obtain the pulse transfer functions by two different methods.

2. Derive the state space model of discrete control system using direct programming method and draw its block diagram.

3. Investigate the controllability and observability of the following system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

4. (a) What are the properties of dead beat response system?
   (b) Explain in detail, how a digital controller for deadbeat performance can be obtained.

5. (a) State the necessary and sufficient conditions for the arbitrary pole placement.
   (b) Explain Ackermoon's formula used for the determination of the state feedback gain matrix K.

6. (a) What are the major theoretical approaches for design of optimal control. Explain one of the approaches in detail.
   (b) Find the extremals for the functional
   $J(\mathrm{x}) = \int_0^{t_1} (\frac{\sqrt{1+\dot{x}^2}}{x})dt \qquad \mathrm{x}(0) = 0;\ \mathrm{x}(t_1) = t_1 - 5.$

7. (a) Explain the summary of the procedure for solving optimal control problems using Hamiltonian Formulation of Variational Calculus.
   (b) Find the curve with the minimum arc length joining the point (0, 0) and the line $\theta(\mathrm{t}) = 2 - \mathrm{t}$.

8. Break up the following transfer matrices into R(s) and P(s)

(a) $T(s) = R(s)P^{-1}(s)$

(b) R(s) and P(s) are relatively right prime,

(c) P(s) is column proper

i. $T(s) = \begin{bmatrix} \frac{s+1}{s^2} & \frac{s+2}{s^2+1} \\ \\ \frac{2}{s} & \frac{2s+3}{s^2+1} \\ \end{bmatrix}$

ii. $T(s) = \begin{bmatrix} \frac{(s-2)(s+1)}{s(s-1)^2} & \frac{1}{(s-1)^2} \\ \\ -\frac{1}{s} & 0 \\ \\ \frac{2}{s(s-1)} & \frac{1}{s-1} \\ \end{bmatrix}$

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1. Consider the difference equation
   y(k+2) - y(k+1) - y(k) = 0; with y(0) = 0, y(1) = 1
   Obtain the general solution y(k) in a closed form and hence show that
   $\underset{k\to\infty}{Lt} \frac{y(k+1)}{y(k)} = \frac{1+\sqrt{5}}{2}$

2. Derive the state space model of discrete control system using direct programming method and draw its block diagram.

3. Determine the stability of the following characteristic equations by using suitable tests.
   (a) $5z^2$- 2z + 2 = 0
   (b) z^3 - 0.2 z^2 - 0.25z + 0.05 = 0
   (c) z^4 - 1.7z^3 + 1.04z^2 - 0.268 z + 0.024 = 0

4. Show that the transfer function U(s) / E(s) of the PID controller shown below.

$$\frac{U(s)}{E(s)} = K_o \frac{T_1 + T_2}{T_1}\left[1 + \frac{1}{(T_1 + T_2)} + \left(\frac{T_1 T_2 \cdot s}{T_1 + T_2}\right)\right]$$

   Assume the gain **k** is very large compared with unity, or **k** >>**1**as shown in the figure 2

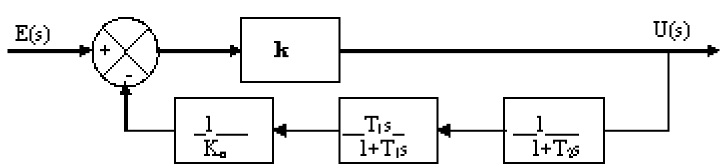

Figure 2:

5. (a) Enumerate the design steps for pole placement.
   (b) Prove Ackermann's formula for the determination of the state feedback gain matrix K.

6. (a) State and explain the minimum - time problems. Describe its performance index.

(b) Let f(X) = -x_1x_2 and let g(X) =$x_1^2 + x_2^2 - 1$. What are the potential candidates for minima of f subject to the constraint g=0?. Show that the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$actually provide the minima.

7. Suppose that the system
$\dot{x}_1(t) = x_2(t)$
$\dot{x}_2(t) = u(t)$
is to be controlled to minimize the performance measure
$J(x, u) = \frac{1}{2}\int\limits_0^2 u^2 dt$
Find a set of necessary conditions for solving optimal control using Hamiltonian Formula of Variational Calculus.

8. (a) With step-by-step procedure explain the observable realization algorithm.

(b) The transfer matrix of a system is

$$T(s) = R(s)P^{-1}(s) = \begin{bmatrix} s+1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s^2 & 0 \\ -1 & s-1 \end{bmatrix}^{-1}$$

Obtain controllable realization in the state space form. Is the realization expected to be minimal?

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