

IV B.Tech I Semester Supplementary Examinations, April/May 2005
DIGITAL CONTROL SYSTEMS
 (Common to Electronics & Instrumentation Engineering and Electronics &
 Control Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
 All Questions carry equal marks

1. Given the discrete-time system

$$y(k) - \frac{1}{\sqrt{2}}y(k-1) + \frac{1}{4}y(k-2) = u(k) + \frac{1}{3}u(k-2)$$

Determine the pulse transfer function and the unit pulse response.

2. Consider the following system given in the controllable canonical form

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [b_3 - a_3 \quad b_0 \quad b_2 - a_2 \quad b_0 \quad b_1 - a_1 \quad b_0] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + b_0 u(k)$$

It is desired to transfer the system equation into the observable canonical form by means of the transformation of the state vector. Determine a transformation matrix T that will give the desired observable canonical form.

3. Investigate the controllability and observability of the following system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

4. Determine the stability of the following characteristic equations by using suitable tests.

(a) $5z^2 - 2z + 2 = 0$

(b) $z^3 - 0.2z^2 - 0.25z + 0.05 = 0$

(c) $z^4 - 1.7z^3 + 1.04z^2 - 0.268z + 0.024 = 0$

5. (a) What are the two basic transformations used to convert an analog system transfer function to a digital system transfer function? Explain each procedure.
- (b) Explain the design of digital controllers through bilinear transformation.

6. Consider the system defined by

$$\dot{X} = Ax + Bu$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

by using the state feedback control $u = -Kx$, it is desired to have the closed loop poles at $s = -2 \pm j4$ and $s = -10$. Determine the state feedback gain matrix K .

7. The plant model of a servo system is given as

$$X(k+1) = GX(k) + Hu(k) \quad Y(k) = CX(k)$$

$$\text{where } G = \begin{bmatrix} 1 & 0.0952 \\ 0.0 & 0.905 \end{bmatrix}; H = \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}; C = [1 \ 0]$$

Find the state feedback control law for the system that places the closed loops at $z_{1,2} = 0.888 \pm j0.173$. The control law is to be realized using a state observer. Choosing the roots of the characteristic equation of the observer as $z = 0.82$, design the observer.

8. Consider the mixing tank system, $x(k+1) = Fx(k) + Gu(k); y(k) = Cx(k)$

$$\text{Where } F = \begin{bmatrix} 0.9512 & 0 \\ 0 & 0.90480 \end{bmatrix}; G = \begin{bmatrix} 4.88 & 4.88 \\ -0.019 & 0.0095 \end{bmatrix}; C = \begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}$$

Find the optimal control law that minimizes the performance index

$$J = \sum_{k=0}^{\infty} [y^T(k) Q y(k) + u^T(k)],$$

$$\text{where } Q = \begin{bmatrix} 25 & 0 \\ 0 & 0.1 \end{bmatrix}; R = \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix}$$
