Code No: RR10102 **Set No. 1**

I B.Tech Supplementary Examinations, November/December 2005
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering, Aeronautical Engineering, Instrumentation & Control Engineering and Bio-Technology)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{x^{2n}}{(n+1)\sqrt{n}}$ [5]
   (b) Test for absolute/ conditional convergence
   $\frac{1}{2}\cdot\frac{1}{1^3} - \frac{2}{3}\cdot\frac{1}{2^3} + \frac{3}{4}\cdot\frac{1}{3^3} - \frac{4}{5}\cdot\frac{1}{4^3} + ...$ [5]
   (c) Verify Roll's theorem for f(x) = log $\left[\frac{x^2+ab}{x(a+b)}\right]$ in [a,b] (x $\neq$ 0) [6]

2. (a) If $\mu = \sin^{-1} \left[\frac{x^{1/3}+y^{1/3}}{\sqrt{x}+\sqrt{y}}\right]^{1/2}$ show that $x\frac{\partial\mu}{\partial x} + y\frac{\partial\mu}{\partial y} = -\frac{1}{12}\tan\mu$
   and $x^2\frac{\partial^2\mu}{\partial x^2} + 2xy\frac{\partial^2\mu}{\partial x\partial y} + y^2\frac{\partial^2\mu}{\partial y^2} = \frac{\tan\mu}{144}(13+\tan^2\mu)$
   (b) Find the evolute of the curve x = a $\mathrm{Cos}^3\theta$, y = a $\sin^3\theta$. [8+8]

3. (a) Trace the Folium of Decartes : $x^3 + y^3 = 3axy$.
   (b) Determine the volume of the solid generated by revolving the limacon
   r = a + b cosθ (a>b) about the initial line. [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant: $x^2+y^2 = c$. [3]
   (b) Solve the differential equation: $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1 + x)\, e^x \sec y$. [7]
   (c) Find the orthogonal trajectories of the coaxial circles $x^2 + y^2 + 2\lambda y + c = 2$, λ being a parameters. [6]

5. (a) Solve the differential equation: y″ - 4y′ + 3y = $4e^{3x}$, y(0) = - 1, y′(0) = 3.
   (b) Solve $(D^2 + 4)y = sec2x$ by the method of variation of parameters. [8+8]

6. (a) State and prove second shifting theorem. [5]
   (b) Find the inverse Laplace Transformation of $\left[\frac{s+3}{(s^2+6s+13)^2}\right]$ [5]
   (c) Evaluate $\int\int\int z^2$dxdydz taken over the volume bounded by $x^2 + y^2 = a^2$, $x^2 + y^2 = z$ and z = 0. [6]

7. (a) Find $\overline{A}.\nabla\phi$ at (1, -1, 1) if $\overline{A}$ =$3xyz^2i + 2xy^3j - x^2yzk$ and $\phi = 3x^2 - yz$.

   (b) Show that $\mathbf{F}$=$(2xy + z^3)i + x^2j + 3xz^2k$ is a conservative force field. Find the scalar potential. Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4). [8+8]

8. (a) Apply Stoke's theorem to evaluate $\int\limits_C (x + y)dx\ +\ (2x - 3)\,dy\ +\ (y + z)dz$where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).

   (b) Evaluate by Green's theorem $\oint\limits_C (Cos\,x\ Sin\,y\ -\ 2\,x\,y\,)\,dx\ +\ Sinx\,Cosy\,dy$ where 'C' is the circle $x^2 + y^2 = 1$. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the series $\sum\limits_{n=1}^{\infty} \frac{1}{2^n+3^n}$ [5]

   (b) Find the interval of convergence of the series $\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + ...\infty$ [5]

   (c) Verify Rolls theorem for f(x) = $2x^3 + x^2 - 4x - 2$ in $\left[-\sqrt{2}, \sqrt{2}\right]$ [6]

2. (a) Expand f(x,y) = $e^y\log(1+x)$ in powers of x and y.

   (b) Show that the evolute of $x = a\left(Cos\theta + \log\tan\frac{\theta}{2}\right)$, y = a Sinθ is the catenary $y = a\,Cos\,h\,\frac{x}{a}$ [8+8]

3. Trace the lemniscate of Bernouli : $r^2 = a^2 \cos 2\theta$. Prove that the volume of revolution about the initial line is $\frac{\pi a^3}{6\sqrt{2}}\left[3\ \log\left(\sqrt{2}+1\right) - \sqrt{2}\right]$ [16]

4. (a) Form the differential equation by eliminating the arbitrary constant $\sin\sqrt{x} + e^{1/y} = c$. [3]

   (b) Solve the differential equation: $\frac{dy}{dx} = \frac{x - y\cos x}{1+\sin x}$ [6]

   (c) The temperature of the body drops from 100^0 C to 75^0C in ten minutes when the surrounding air is at 20^0C temperature. What will be its temperature after half an hour. When will the temperature be 25^0C. [6]

5. (a) Solve the differential equation: $(D^2 + 4D + 4)y = 18$ coshx.

   (b) Solve the differential equation: $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$ [8+8]

6. (a) Evaluate L$\{e^t(\cos 2t + {}^1\!/_2 \sinh 2t)\}$ [5]

   (b) Find the inverse Laplace Tranformations of $\left[\frac{4}{(s+1)(s+2)}\right]$ [5]

   (c) Evaluate the integral $\int\int\int xy^2z$ dxdydz taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [6]

7. (a) Prove that $\nabla\left[\nabla\,.\left(\frac{\overline{r}}{r}\right)\right]=-\frac{2\,\overline{r}}{r^3}$

   (b) Find the work done in moving a particle in the force field $\overline{F}=3x^2\,i\;+\;(2xz\;-\;y)\,j\;+\;z\,k$ along the curve $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. [8+8]

8. (a) Apply Green's theorem to evaluate $\oint\limits_C (2xy - x^2)dx + (x^2 + y^2)dy$, where "C" is bounded by $y = x^2$ and $y^2 = x$.

   (b) Apply Stoke's theorem to evaluate $\int\limits_C y\,dx\;+\;z\,dy\;+\;x\,dz$ where 'C' is the curve of the intersection of the sphere$x^2 + y^2 + z^2 = a^2$ and $x + z = a$. [8+8]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test the convergence of the following series $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n+1)}{2.5.8....(3n+2)}$ [5]

   (b) Test the following series for absolute /conditional convergence $\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n^2}$ [5]

   (c) Expand sin x in powers of $\left(x - \Pi/2\right)$ up to the term containing $\left(x - \Pi/2\right)^4$ [6]

2. (a) If U=Sin$^{-1}\left(\frac{x}{y}\right)$+ tan^{-1} (y/x) show that $x\,\frac{\partial \mu}{\partial x} + y\,\frac{\partial \mu}{\partial y} = 0$ [6]

   (b) Find the radius of curvature at any point on the curve y = c cosh$\frac{x}{c}$ [10]

3. Trace the curve : r = a (1 + cos θ). Show that the volume of revolution of it about the initial line is 8πa^3/ 3. [16]

4. (a) Form the differential equation by eliminating the arbitrary constant sin^{-1}x + sin^{-1}y = c. [3]

   (b) Solve the differential equation: coshx $\frac{dy}{dx}$ + y sinhx = 2 cosh^{2}x sinhx [7]

   (c) Find the orthogonal trajectories of the family: r^n sin nθ = b^n. [6]

5. (a) Solve the differential equation: $(D^2 + 4)y$ = sin t + 1/3 sin3t + 1/5 sin5t, y(0) = 1, y ′(0) = 3/35.

   (b) Solve the differential equation: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\,Cos\,\log(1+x)$ [8+8]

6. (a) Find $L\left[\frac{\sin^2 t}{t}\right]$ [5]

   (b) Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$ [6]

(c) Evaluate the triple integral $\int\limits_{0}^{\pi/2} \int\limits_{0}^{a\,Sin\theta} \int\limits_{1}^{\frac{a^2-r^2}{a}} r dz dr d\theta$ [5]

7. (a) Evaluate $\nabla^2 \log r$ where $r = \sqrt{x^2 + y^2 + z^2}$

   (b) Find constants a, b, c so that the vector $\mathbf{A}$ =(x+2y+az)$\mathbf{i}$ +(bx-3y-z)$\mathbf{j}$+(4x+cy+2z)$\mathbf{k}$ is irrotational. Also find φ such that $\mathbf{A} = \nabla\phi$. [8+8]

8. Verify divergence theorem for F = 6zi + (2x + y)j − xk, taken over the region bounded by the surface of the cylinder $x^2 + y^2 = 9$ included in z = 0, z = 8, x = 0 and y = 0. [16]

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Test the convergence of the series $\frac{\sqrt{2}-1}{3^2-1}+\frac{\sqrt{3}-1}{4^2-1}+\frac{\sqrt{4}-1}{5^2-1}+\ldots.$ [5]
   (b) Test whether the following series converges absolutely or conditionally. $1.\frac{2}{3}-\frac{1}{2}.\frac{3}{4}+\frac{1}{3}.\frac{4}{5}-\frac{1.5}{4.6}+\ldots..$ [5]
   (c) Verify Roll's theorem for f(x) = x (x+3) $e^{-x/2}$ in [-3, 0] [6]

2. (a) If $x^x y^y z^z = c$, then show that at x = y = z, $\frac{\partial^2 z}{\partial x \partial y} = -\{x \log(ex)\}^{-1}$
   (b) Find the evolute of the hyperbola $x^2/a^2 - y^2/b^2 = 1$. Deduce the evolute of a rectangular hyperbola. [8+8]

3. (a) Trace the curve $a^2y^2 = x^2(a^2 - x^2)$
   (b) Find the length of the arc of the curve $y = \log\left[\frac{e^x-1}{e^x+1}\right]$ from x = 1 to x = 2 [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant y sec x = c + $x^2/2$. [3]
   (b) Find the orthogonal trajectories to $x^2 - y^2 = a^2$. [7]
   (c) A body heated to 110^0 C is placed in air at 10^0 C. After 1 hour its temperature is 80^0 C. When the temperature will be 30^0 C. ? [6]

5. (a) Solve the differential equation: $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = Sin\,2x$
   (b) Solve the differential equation: $(D-2)^2 = 8(e^{2x} + sin2x + x^2)$ [8+8]

6. (a) Show that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\left[\overline{f}(s)\right]$ where n = 1,2,3, ... [5]
   (b) Find $L^{-1}\{s / (s^2 - a^2)\}$ [5]
   (c) Evaluate: $\int\limits_0^1 \int\limits_0^{\sqrt{1+x^2}} \frac{dx\,dy}{(1+x^2+y^2)}$ [6]

7. (a) Evaluate $\nabla.[r\nabla(1/r^3)]$ where $r = \sqrt{x^2+y^2+z^2}$

(b) Evaluate $\iint_s \mathbf{A}.\mathbf{n}$ ds where $\mathbf{A}$=18zi-12j+3yk and s is that part of the plane 2x+3y +6z=12 which is located in the first octant. [8+8]

8. State Green's theorem and verify Green's theorem for $\oint_C [(xy + y^2)dx + x^2dy]$, where c is bounded by y = x and y = x^2. [16]

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