Code No: NR10102 **NR**

I B.Tech Supplementary Examinations, November/December 2005
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Computer Science & Engineering, Chemical Engineering, Electronics & Instrumentation Engineering, Bio-Medical Engineering, Information Technology, Electronics & Control Engineering, Mechatronics, Computer Science & Systems Engineering, Electronics & Telematics, Metallurgy & Material Technology, Electronics & Computer Engineering, Production Engineering and Aeronautical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Test for convergence of the series $\sum_{\infty}^{1}\left[\sqrt{n^4+1}\ -\ \sqrt{n^4-1}\right]$ [5]
   (b) State and prove Cauchy Mean value theorem. [5]
   (c) If a < b prove that $\frac{b-a}{(1+b^2)} < tan^{-1}b - tan^{-1}a < \frac{b-a}{(1+a^2)}$ using Lagrange's Mean value theorem. Deduce the following [6]
      i. $\frac{\Pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\Pi}{4} + \frac{1}{6}$
      ii. $\frac{5\Pi+4}{20} < \tan^{-1}2 < \frac{\Pi+2}{4}$

2. (a) If $\mu = \log\ (x^3+y^3+z^3-3xyz)$ prove that
   $\mu_x + \mu_y + \mu_z = 3(x+y+z)^{-1}$ and $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 \mu = \frac{-9}{(x+y+z)^2}$
   (b) Find the radius of curvature at any point of the parabola $y^2 = 4ax$. Prove that the square of the radius of curvature at any point of the curve varies as the cube of the focal distance of the point. [8+8]

3. (a) Trace the curve $9ay^2 = x\ (x-3a)^2$.
   (b) Prove that the length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum is $2a[\sqrt{2} + \log\ (\ 1 + \sqrt{2})]$ [8+8]

4. (a) Form the differential equation by eliminating the arbitrary constant $y = \frac{a+x}{x^2+1}$. [3]
   (b) Solve the differential equation: $x\frac{dy}{dx} + y = x^3y^6$. [7]
   (c) The temperature of cup of coffee is 92^0 C, when freshly poured the room temperature being 24^0 C. In one minute is was cooled to 80^0 C. how long a period must elapse, before the temperature of the cup becomes 65^0 C. [6]

5. (a) Solve the differential equation: $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = Sin\,2x$
   (b) Solve the differential equation: $(D-2)^2 = 8(e^{2x} + sin2x + x^2)$ [8+8]

6. (a) Evaluate $L\{e^t(\cos 2t + \frac{1}{2}\sinh 2t)\}$ [5]

(b) Find the inverse Laplace Tranformations of $\left[\frac{4}{(s+1)(s+2)}\right]$ [5]

(c) Evaluate the integral $\int\int\int xy^2z\ dxdydz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$. [6]

7. Prove that $\mathbf{F}=(y^2\cos x + z^3)i + (2y\sin x - 4)j + (3xz^2 + 2)k$ is a conservative force field. Find the work done in moving an object in this field from (0, 1, -1) to (π/2, -1, 2). [16]

8. Verify divergence theorem for $2x^2y\mathbf{i} - y^2\mathbf{j} + 4xz^2\mathbf{k}$ taken over the region of first octant of the cylinder $y^2+z^2 = 9$ and $x = 2$. [16]

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