Code No: RR210403 **Set No.1**

II B.Tech. I Semester Regular Examinations, November -2005
PROBABILITY THEORY & STOCHASTIC PROCESS
(Common to Electronics & Communication Engineering and Electronics & Telematics)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) List and explain the properties of probability density function
   (b) Three newspapers A, B and C are published in a city and a survey of readers indicates the following:
   20% read A, 16% read B, 14% read C, 8% read A and B, 5% read A and C, 2% read A, B and C.
   For one adult chosen at random, compute the probability that:
      i. he reads none of the papers.
      ii. he reads exactly one of the papers
      iii. he reads at least A and B if it is known that he reads at least one of the papers published.

   [7+9]

2. (a) A fair coin is tossed three times and the faces showing up are observed
      i. Write the Sample description space
      ii. If X is the number of heads in each of the outcomes of this experiment, find the probability function.
      iii. Sketch the CDF and pdf.
   (b) The continuous random variable X has a p.d.f. $f(x) = x/2,\ 0 \leq x \leq 2$. Two independent determinations of X are made. What is the probability that both these determinations will be greater than one. If three independent determinations are made, what is the probability that exactly two of these are larger than one?

   [10+6]

3. (a) State and prove central limit theorem
   (b) If $f_{X,Y}(X,Y) = 0.5\exp(-|X| - |Y|)$, where X and y are two random variable, if Z=X+Y , find $f_Z(Z)$

   [8+8]

4. Consider a Random binary waveform that consists of a sequence of pulses with the following properties
   (a) Each pulse is of duration T_0
   (b) Pulses are Equally likely to be ± 1

(c) All pulses are statistically independent

(d) The pulses are not synchronized, that is, the starting time T of the first pulse is Equally likely to be anywhere between 0 and Tb

Find the Auto correlation and power spectral density function of x(t). [8+8]

5. Find the input auto correlation function, output autocorrelation and o/p spectral density of RC low pass filter, where the filter is subjected to a white noise of spectral density $N_0/2$. [16]

6. (a) Explain how partition noise is present in electron devices?

(b) Explain the usefulness of knowing the noise power spectral density of a network.

[8+8]

7. (a) Discuss the significance of noise equivalent temperature of an electronic system.

(b) Evaluate the equivalent noise temperature of a two port device with a matched source and a matched load.

[8+8]

8. (a) A source emits seven messages with probabilities 1/2, 1/4, 1/8, 1/16, 1/32, 1/64 and 1/64 respectively. Find the entropy of the source. Obtain the compact binary code and find the average length of the code word. Determine the efficiency and redundancy of the code.

(b) Consider AWGN channel with B = 3 KHz, find the minimum value of S/N in dB for reliable information transmission at R = 2400, 4800 and 9600 bit / sec.

[10+6]

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1. (a) State and prove Bayes theorem of probability.
   (b) In a single throw of two dice, what is the probability of obtaining a sum of at least 10?

   [8+8]

2. (a) A fair coin is tossed three times and the faces showing up are observed
      i. Write the Sample description space
      ii. If X is the number of heads in each of the outcomes of this experiment, find the probability function.
      iii. Sketch the CDF and pdf.
   (b) The continuous random variable X has a p.d.f. f(x) = x/2, $0 \leq x \leq 2$. Two independent determinations of X are made. What is the probability that both these determinations will be greater than one. If three independent determinations are made, what is the probability that exactly two of these are larger than one?

   [10+6]

3. (a) A fair Coin is tossed three times. Let X be the number of tails appearing. Find the probability distribution of X calculate the expected value of X.
   (b) Show that $E[X-m]^3 = E(X)^3 - 3m_x\sigma_x^2 - m_x^3$ where m_x and σ_x^2 are the mean and variance of X respectively.

   [8+8]

4. Consider a Random binary waveform that consists of a sequence of pulses with the following properties
   (a) Each pulse is of duration T_0
   (b) Pulses are Equally likely to be ± 1
   (c) All pulses are statistically independent
   (d) The pulses are not synchronized, that is, the starting time T of the first pulse is Equally likely to be anywhere between 0 and Tb

   Find the Auto correlation and power spectral density function of x(t). [8+8]

5. (a) Find the PSD of a random process $z(t) = X(t) + y(t)$ where x(t) and y(t) are zero mean, individual random process.

   (b) A wss random process x(t) is applied to the input of an LTI system whose impulse response is $5t.e^{-2t}$ The mean of x(t) is 3. Find the output of the system.

[8+8]

6. (a) Explain available power of a noise source.

   (b) Explain the components of noise power spectral density.

[8+8]

7. (a) Derive the mathematical description of noise figure.

   (b) An amplifier with $g_a = 40$dB and $B_N = 20$KHz is found to have $N_{ao} = 10K\ T_0$ when $T_i = T_0$. Find Te and noise figure.

[10+6]

8. (a) Write a short notes on the role and use of Information theory to a communication engineer.

   (b) A card is drawn at random from ordinary deck of 52 playing cards. Find the information in bits that you receive when you are told that the card is a heart, a face card, and a heart face card.

[8+8]

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1. (a) Give the classical and axiomatic definitions of Probability.
   (b) If a three digit decimal number is chosen at random, find the probability that exactly K digits are greater than equal to 5, for $0 \leq K \leq 3$.
   (c) Three boxes of identical appearance contain two coins each. In one box both are gold; in the second both are silver and in the third box one is silver and the other is the gold coin. Suppose that a box is selected at random and further that a coin in that box is selected at random. If this coin proves to be gold, what is the probability that the other coin is also gold?

   [4+6+6]

2. (a) Define Conditional Probability mass function.
   (b) If two random variables have the joint probability density

   $$f(x_1, x_2) = \frac{2}{3}(x_1 + 2x_2) \qquad \text{for } 0 < x1 < 1, 0 < x2 < 1$$
   $$= 0 \text{ else where}$$

   Find
   i. the marginal density of x_2.
   ii. Conditional density of the first given that the second takes on the value x_2.
   (c) A pair of dice is tossed. Define a random variable X to be the difference of the face values turned up. Determine the probability mass function of X

   [4+6+6]

3. (a) For a function Y=$(X - m_x)/\sigma_x$, prove that mean is zero & variance is 1
   (b) For the joint distribution of (X,Y) given by
   $f_{xy}(x, y) = \frac{1}{4a^2}\left[(1 + xy)\left(x^2 - y^2\right], |x| <= a, |y| <= a, a > 0\right.$
   $= 0$, otherwise
   Show that the Characteristic function of X+Y is equal to the product of the characteristic function of X & Y.

   [8+8]

4. (a) If the auto correlation function of a wss process is $R(\tau) = k\ .e^{-k(\tau)}$, show that its spectral density is given by $S(\omega) = \frac{2}{1+\left(\frac{\omega}{k}\right)^2}$

(b) Find the PSD of a random process x(t) if E[x(t)] = 1 and $R_{xx}(\tau) = 1 + e^{-\alpha|\tau|}$

[8+8]

5. White noise n(t) with G(f) =$\eta/2$ is passed through a low pass RC network with a 3dB frequency f_c.

   (a) Find the autocorrelation R(τ) of the out put noise of the network.

   (b) Sketch P(τ) =$R(\tau)/R(0)$

   (c) Find $\varpi_c(\tau)$such that $P(\tau) \leq$0.1.

[8+4+4]

6. Give reasons for the following:

   (a) In any communication system the first stage must have low noise operation.

   (b) Describe how FET gives low noise performance compared to BJT.

[8+8]

7. (a) Define noise temperature and noise figure.

   (b) Show that the noise figure of a well designed receiver is usually near about the noise figure of the low noise RF stage amplifier at the antenna input.

[8+8]

8. (a) Define the terms: Information, Self Information and Information rate with respect to a source.

   (b) A source generates eight symbols: A, B, C, D, E, F, G and H with probabilities 0.5, 0.2, 0.1, 0.05, 0.05, 0.05, 0.03 and 0.02. Find the entropy and information rate if the symbols are generated at a rate of 1000 / Sec.

[6+10]

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1. (a) State and derive the theorem on Total probability.
   (b) A shipment of components consists of three identical boxes. One box contains 2000 components of which 25% are defective, the second box has 5000 components of which 20% are defective and the third box contains 2000 components of which 600 are defective. A box is selected at random and a component is removed at random from the box. What is the probability that this component is defective? What is the probability that it came from the second box?

   [8+8]

2. (a) Define joint distribution and Joint probability density function for the two random variables X and Y.
   (b) Let X and Y be jointly continuous random variables with joint density function

$$f(x,y) = xy\exp^{\left[-\frac{+}{2}(x_2+y_2)\right]}; \qquad x>0, y>0$$

$$= 0 \quad \text{otherwise}$$

   Check whether X and Y are independent. Find
   i. $P(X \leq 1, Y \leq 1)$ and
   ii. $P(X+Y \leq 1)$

   [4+12]

3. (a) If X is random variable, show that var(aX+b) = a^2 var(X)
   (b) A random variable z is uniformly distributed having Probability density function
   $f_Z(z) = 1/2, \qquad -1 <= Z <= 1$
   $= 0$, otherwise
   Show that the random variables X=Z and Y=Z^2 are un-correlated despite of the fact that they are statistically dependant.

   [6+10]

4. Consider a Random binary waveform that consists of a sequence of pulses with the following properties
   (a) Each pulse is of duration T_0

(b) Pulses are Equally likely to be ± 1

(c) All pulses are statistically independent

(d) The pulses are not synchronized, that is, the starting time T of the first pulse is Equally likely to be anywhere between 0 and Tb

Find the Auto correlation and power spectral density function of x(t). [8+8]

5. (a) Derive the relation between PSDs of input and output random process of an LTI system.

(b) X(t) is a stationary random process with zero mean and auto correlation $R_{XX}(\tau)\, e^{-2|\tau|}$is applied to a system of function $H(w) = \frac{1}{jw+2}$ Find mean and PSD of its output.

[8+8]

6. (a) What are the characteristics of shot noise?

(b) What are the important requirements of the front-end stage of a communication receiver in the point of view of noise?

[8+8]

7. (a) An amplifier has input and output impedances of 75 ohm, 60dB power gain, and a noise equivalent bandwidth of 15KHz. When a 75Ω resistor at $290K$ is connected to the input, the output rms noise voltage is 75microvolt. Determine the effective noise temperature of the amplifier assuming that the meter is impedance matched to the amplifier.

(b) List the devices in which narrowband noise can be present.

[8+8]

8. (a) Estimate the communication rate of a binary symmetric system in which the two symbols A and B were transmitted at a rate of 1000 symbols / sec, if the system noise cause 1 % of the symbols to be received incorrectly.

(b) Determine the maximum information rate in case of a Teletype writer system capable of,

i. A rate of 100 words per minute,

ii. A radio speech communication system with a bandwidth of 5000 Hz and a signal to noise power ratio of 30 dB.

[8+8]

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