Code No: NR220202 **NR**

II B.Tech II Semester Supplementary Examinations, November/December 2005
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Mechanical Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Mechatronics, Electronics & Telematics, Metallurgy & Material Technology and Aeronautical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. Evaluate the following using $\beta - \Gamma$ functions
   (a) $\int_0^1 x^7(1-x)^5 dx$
   (b) $\int_0^{\pi/2} \sin^5\theta \cos^{7/2}\theta d\theta$
   (c) $\int_0^\infty y^{-1/2}(1-e^{-y})dy$ [5+5+6]

2. Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$ [16]

3. (a) If w = u + iv is an analytic function of z and u + v = $\frac{\sin 2x}{\cos h\, 2y - \cos 2x}$ then find f(z)
   (b) If sin $(\theta + i\phi)$ = cos α + i sin α, then prove that $\cos^2\theta = \sin^2\alpha$ [8+8]

4. (a) Evaluate $\int_c \frac{z^3 - \sin 3z\ dz}{(z-\frac{\pi}{2})^3}$ with c: $|z| = 2$ using Cauchy's integral formula
   (b) Evaluate $\int_C (z^2 + 3z + 2)$where C is the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between the points (0,0) to ($\prod$a, 2a) [8+8]

5. Expand $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$ in the region
   (i) 1 < | z | < 4
   (ii) | z | < 1. [8+8]

6. (a) Find the poles and residues at each pole Tanh z
   (b) Evaluate $\int_C \frac{z^3\ dz}{(3-1)^2(z-3)}$ where c is $|z| = 2$ by residue theorem [8+8]

7. (a) Evaluate $\int_0^\pi \frac{(1+2\cos\theta)d\theta}{(5+4\cos\theta)}$ using residue theorem.
   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)^2}$, a>0 using residue theorem [8+8]

8. (a) Find and plot the rectangular region $0 \leq x \leq 1$; $0 \leq y \leq 1$, under the transformation $w = \sqrt{2}\, e^{i\pi/4} z + (1-2i)$.

   (b) Show that the map of the real axis of the z-plane on to the w-plane by the transformation $W = \frac{1}{z} + i$ is a circle. Find its center and radius. [8+8]

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