Code No: RR220202

Set No. 1

II B.Tech II Semester Supplementary Examinations, November/December 2005

MATHEMATICS-III

(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Show that $\int_{-1}^{1} (1+x)^{p-1}(1-x)^{q-1}dx = 2^{p+q-1}\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

   (b) Show that $\int_{0}^{1} \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{\beta(m,n)}{a^n(1+a)^m}$ [8+8]

2. (a) Using Rodrigue's formula prove that $\int_{-1}^{1} x^m P_n(x)dx = 0$ if m < n

   (b) Prove that $J_0(x) = \frac{1}{\pi}\int_{0}^{\pi} \cos(x\cos\theta)d\theta$. [8+8]

3. (a) Prove that U = $\overset{-x}{e}$ $[(x^2-y^2)\cos y + 2xy\sin y]$ is harmonic and the analytic function whose real pant is u.

   (b) Separate the real and imaginary parts of Sin h z. [8+8]

4. (a) Evaluate $\int_{c} \frac{Cos\, z - \sin\, z\;\; dz}{(z+i)^3}$ with c: | z | = 2 using Cauchy's integral formula

   (b) Evaluate $\int_{1-i}^{2+i} 2x+1+iy)dz$ along (1-i) to (2+i) using Cauchy's integral formula [8+8]

5. (a) Find the Laurent series expansion of the function. $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3 < | z + 2 | < 5.

   (b) Obtain the Taylor series expansion of $f(z) = \frac{e^z}{z(z+1)}$ about z = 2. [8+8]

6. (a) State and prove Cauchy's Residue Theorem.

   (b) Find the residue of $\frac{Z^2}{Z^4-1}$at these Singular points which lie inside the circle | z | = 2. [8+8]

7. (a) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2}$ using residue theorem.

   (b) Evaluate$\int_{0}^{\infty} \frac{x\ \sin\ mx\, dx}{x^4+16}$ using residue theorem. [8+8]

8. (a) Define conformal mapping? Let f(z) be analytic function of z in a domain D of the z-plane and let $f'(z) \neq 0$ in D. Then show that $w=f(z)$ is a conformal mapping at all points of D.

   (b) Find the bilinear transformation which maps the points (–i, o, i) into the point (–l, i, l) respectively. [8+8]

⋆⋆⋆⋆⋆

Code No: RR220202 **Set No. 2**

II B.Tech II Semester Supplementary Examinations,November/December 2005

MATHEMATICS-III

(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Show that β(m,n)=2 $\int\limits_{o}^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta d\theta$ and deduce that $\int\limits_{o}^{\pi/2} \sin^{n}\theta d\theta = \int\limits_{o}^{\pi/2} \cos^{n}\theta d\theta = \frac{\Gamma(n+1)/2)\Gamma\left(\frac{1}{2}\right)}{2\Gamma(\frac{1}{2}(n+2))}$

   (b) Prove that Γ(n)Γ(1-n)= $\frac{\prod}{\sin n \prod}$

   (c) Show that $\int\limits_{0}^{\infty} x^m e^{-a^2x^2}dx = \frac{1}{2a^{n+1}}\Gamma(n+1)/2$ and hence deduce that $\int\limits_{o}^{\infty} \cos(x^2)dx = \int\limits_{o}^{\infty} \sin(x^2)dx = 1/2\sqrt{\pi/2}$ [5+5+6]

2. (a) Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0$(x) + P_1(x) t + P_2(x) t^2+ ....

   (b) Write $J_{5/2}$(x) in finite form. [8+8]

3. (a) Find the analytic function f(z) = u + iv if u-v= e^x(cos y-sin y)

   (b) Find all principal values of $(1 + i\sqrt{3})^{1+i\sqrt{3}}$ [8+8]

4. (a) Evaluate $\int\limits_{c} \frac{z^3-\sin 3z \ \ dz}{(z-\frac{\pi}{2})^3}$ with c: $|\,z\,| = 2$ using Cauchy's integral formula

   (b) Evaluate $\int\limits_{C} (z^2 + 3z + 2)$where C is the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a\,(1 - \cos\theta)$ between the points (0,0) to ($\prod$a, 2a) [8+8]

5. (a) Find the Laurent series expansion of the function. $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3 < | z + 2 | < 5.

   (b) Obtain the Taylor series expansion of $f(z) = \frac{e^z}{z(z+1)}$ about z = 2. [8+8]

6. (a) Find the poles and residues at each pole $\frac{ze^z}{(z-1)^3}$

   (b) Evaluate $\int\limits_{C} \frac{2e^z dz}{z(z-3)}$ where C is |z| = 2 by residue theorem. [8+8]

7. (a) Show that $\int\limits_{0}^{2\pi} \frac{d\theta}{a+bSin\theta} = \int\limits_{0}^{2\pi} \frac{d\theta}{a+bSin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, a > b> 0 using residue theorem.

(b) Evaluate by contour integration $\int\limits_{0}^{\infty} \frac{dx}{1+x^2}$ [8+8]

8. (a) Show that the transformation w=z^2 maps the circle |z-1|=1 into the cardioid r=2(1+cosθ) where w=r$e^{i\theta}$ in the w-plane.

   (b) Find the bilinear transformation which maps the vertices (1+i, -i, 2-i) of the triangle T of the z-plane into the points (0, 1, i) of the w-plane. [8+8]

⋆⋆⋆⋆⋆

II B.Tech II Semester Supplementary Examinations,November/December 2005

MATHEMATICS-III

(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Evaluate $4\int\limits_0^\infty \frac{x^2dx}{1+x^4}$ using $\beta - \Gamma$ functions
   (b) Prove that $\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\Pi}{m, 2^{4m-1}}$
   (c) Evaluate $\int\limits_0^2 (8-x^3)^{1/3}dx$ using $\beta - \Gamma$functions [5+5+6]

2. Prove that $\int\limits_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \frac{2}{2n+1} & if\ m = n \end{cases}$ [16]

3. (a) Show that the real and imaginary parts of an analytic function f(z) = u(r,θ) + i v(r,θ) satisfy the Laplace equation in polar form $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ and $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} = 0$ respectively
   (b) If u is a harmonic function, show that w = u^2 is not a harmonic function unless u is a constant. [8+8]

4. (a) Evaluate $\int\limits_c \frac{z^2-2z-2 \quad dz}{(z^2+1)^2 z}$ where c is $|\, z - i\,| = 1/2$ using Cauchy's integral formula
   (b) Evaluate $\int\limits_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2)dz$ along y=x^2
   (c) Evaluate $\int\limits_c \frac{e^{2z}\ dz}{(z^2+\Pi^2)^3}$ where c is $|\, z\,| = 4$ using Cauchy's integral formula [5+5+6]

5. (a) Find the Laurent series of the functions $f(z) = \frac{z}{(z+1)(z+2),}$ about z = - 2
   (b) Expand f(z) = sinz in Taylor's Series about z = $\Pi/4$. . [8+8]

6. (a) Find the poles and the residues at each pole of f(z) = $\frac{1}{(z^2+4)^2}$.
   (b) Evaluate $\int\limits_C \frac{(3z-4)}{z(z-1)}dz$ where C is the circle |z| =2 using residue theorem [8+8]

7. (a) Evaluate State and prove Rouche's theorem
   (b) Evaluate $\int\limits_0^{2\pi} \frac{\sin 3\theta\, d\theta}{5-3\cos\theta}$ using residue theorem [8+8]

8. (a) Discuss the transformation w=cos z.

   (b) Find the bilinear transformation which maps the points (l, i, -l) into the points (o,1,∞). [8+8]

⋆ ⋆ ⋆ ⋆ ⋆

Code No: RR220202

Set No. 4

II B.Tech II Semester Supplementary Examinations, November/December 2005

MATHEMATICS-III

(Common to Electrical & Electronic Engineering, Electronics & Communication Engineering, Electronics & Instrumentation Engineering, Electronics & Control Engineering, Electronics & Telematics, Metallurgy & Material Technology, Aeronautical Engineering and Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Show that $\int\limits_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive interger and m>-1
   (b) Show that β(m,n)= $\int\limits_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$
   (c) Show that $\int\limits_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$ [6+5+5]

2. (a) Prove that $\frac{1+z}{z\sqrt{1-2xz+z^2}} - \frac{1}{z} = \sum\limits_{n=0}^{\infty} (P_n(x) + P_{n+1}(x))\, z^n$
   (b) Prove that $\frac{d}{dx} (xJ_n \, J_{n+1}) = x(J_n^2 - J_{n+1}^2)$
   (c) Prove that cos x=J_0-2J_2+ 2J_4......... [6+5+5]

3. (a) State sufficient condition for f(z) to be analytic and prove it.
   (b) Find all principal values of $\left(\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}\right)^{(1+i\sqrt{3}}$ [8+8]

4. (a) Evaluate $\int\limits_c \frac{e^z \sin 2z-1 \quad dz}{z^2(z+2)^2}$ where c is $|\,z\,| = 1/2$ using Cauchy's integral formula
   (b) Evaluate $\int\limits_0^{1+i} (x - y^2 + ix^3)dz$ Along the real axis from z=0 to z=1 using Cauchy's integral formula
   (c) Evaluate $\int\limits_c \frac{e^{-2z} z^2 \quad dz}{(z-1)^3(z+2)}$ where c is $|\,z + 2\,| = 1$ using Cauchy's integral formula [5+5+6]

5. (a) State and prove Taylor's theorem.
   (b) Obtain Taylor series to represent the function $\frac{z^2-1}{(z+2)\,(z+3)}$ in the region $|z| < 2$ [8+8]

6. (a) Find the poles and the residues at each pole of f(z) = $\frac{1}{(z^2+4)^2}$.
   (b) Evaluate $\int\limits_C \frac{(3z-4)}{z(z-1)} dz$ where C is the circle |z| =2 using residue theorem [8+8]

7. (a) Show that $\int\limits_{0}^{\pi} \frac{Cos2\theta}{1-2aCos\theta+a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$, (a^2 $<$ 1) using residue theorem.

   (b) Show by the method of contour integration that $\int\limits_{0}^{\infty} \frac{Cosmx}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3}(1+ma)e^{-ma}$,
   (a $>$ 0 , b $>$ 0). [8+8]

8. (a) Prove that every bilinear transformation maps the totality of the circles and straight lines in z-plane on to the totality of circles and straight lines in the w-plane.

   (b) Find the bilinear transformation that maps the points 0,i,1 into the points. [8+8]

⋆ ⋆ ⋆ ⋆ ⋆