

II B.Tech II Semester Supplementary Examinations,
November/December 2005
PROBABILITY AND STATISTICS
(Common to Civil Engineering, Computer Science & Engineering,
Chemical Engineering, Information Technology, Computer Science &
Systems Engineering, Electronics & Computer Engineering and Production
Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) The probability that a regularly scheduled flight departs on time is $P(D) = 0.83$, the probability that it arrives on time is $P(A) = 0.82$ and the probability that it departs and arrives on time is $P(D \cap A) = 0.78$. Find the probability that a plane
- i. arrives on time given that it departed on time and
 - ii. departed on time given that it has arrived on time.

- (b) One bag contains 4 White balls and 3 black balls and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

[8+8]

2. (a) Define Poisson distribution. Find its mean and variance.
- (b) Out of 2000 families with 4 children each, how many would you expect to have
- i. at least one boy
 - ii. exactly two boys
 - iii. no girls.

[8+8]

3. (a) The amount of bread (in hundreds of pounds) X that a certain bakery is able to sell in a day is found to be a random variable, whose density function is given by

$$\begin{aligned}
 f(x) &= Ax \text{ for } 0 \leq x < 5 \\
 &= A(10 - x) \text{ for } 5 \leq x < 10 \\
 &= 0, \text{ otherwise.}
 \end{aligned}$$

Find the value of A such that $f(x)$ is a probability density function. What is the probability that the number of pounds of bread that will be sold tomorrow is

- i. more than 500 pounds
- ii. between 250 and 750 pounds.

- (b) Derive normal distribution as limiting case of binomial distribution, clearly stating the conditions involved.

[8+8]

4. (a) If measurements of the specific gravity of a metal can be looked upon as a sample from a normal population having a standard deviation 0.04, what is the probability that the mean of a random sample of size 25 will be “off” by at most 0.02?
- (b) A normal population has a variance of 15. If the samples of size 5 are drawn from this population, what percentage can be expected to have variances
- less than 10
 - between 5 and 10

[8+8]

5. The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.
- (a) Construct a 90% confidence interval for the mean height of all college students.
- (b) What can we assert with 98% confidence about the possible size of our error if we estimate the mean height of all college students to be 174.5 centimeters?
- (c) A Geiger counter was set up in the physics laboratory to record the number of alpha particle emission of carbon-14 in half a second. Ten observations yielded the following data:

4	6	9	6	10	11	6	3	7	10
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Give the value of an unbiased estimate for λ , the mean number of counts per half-second, assuming that these are observations of a Poisson random variable.

[4+6+6]

6. (a) It is desired to test the hypothesis $\mu_0 = 40$ against the alternative hypothesis $\mu_1 = 42$ on the basis of a random sample from a normal population with the standard deviation $\sigma = 4$. If the probability of a Type I error is to be 0.05 and the probability of a Type II error is to be 0.24, find the required size of the sample.
- (b) The diameter of rotor shafts in a lot has a mean of 0.249 inch and a standard deviation of 0.003 inch. The inner diameters of bearings in another lot have a mean of 0.255 inch and a standard deviation of 0.002 inch. (i) What are the mean and the standard deviation of the clearances between shafts and bearings selected from these lots? (ii) If a shaft and a bearing are selected at random, what is the probability that the shaft will not fit inside the bearing? (Assume that both dimensions are normally distributed)

[8+8]

7. (a) Fit an exponential curve of the $y = Ae^{BX}$ for the following data

x	1	2	3	4
y	7	11	17	27

- (b) Predict y at x= 3.75 by fitting a power curve to the given data

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.10	6.80	7.50

8. Calculate the correlation coefficient r for the following data

x	63	50	55	65	55	70	64	70	58	68	52	60
y	87	74	76	90	85	87	92	98	82	91	77	78

[16]
