

III B.Tech II Semester Supplementary Examinations,
November/December 2005
MATHEMATICAL METHODS FOR CHEMICAL ENGINEERING
(Chemical Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. A tank contains $100ft^3$ of fresh water, $2ft^3$ of brine, having a concentration of 1 pcf of salt, is run into the tank per minute, and the mixture, kept uniform by mixing, runs out at the rate of $1ft^3/min$. What will be the exit brine concentration when the tank contains $150ft^3$ of brine? [16]
2. (a) Discuss the general problem that arises constantly in chemical Engineering.
(b) The fundamental laws governing each of the unit operations in chemical engineering and most readily expressed in the form of simple rate equations. Discuss. [8+8]
3. If the three thermodynamic variables P, V, and T are connected by a relation $(P,V,T) = 0$ show that :

$$\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$
 [16]
4. (a) Discuss in detail the usefulness of partial differential equations in solving chemical engineering problems with a specific problem and with its solution.
(b) Find du/dx if $f(x,y) = 3x^3y^2 + x\cos y$
 where $\frac{\partial f}{\partial x} = 9x^2 + \cos y$
 $\frac{\partial f}{\partial y} = 6x^2 - x \sin y$ [6+10]
5. If A and B are non collinear vectors and $P = (2x - 3y - z) A + (3x + 2y + 5) B$ and $Q = (-x + 4y - z) A + (3x - 4y + 7) B$, find x, y such that $7P = 3Q$. [16]
6. Derive the diffusion equation for a binary perfect gas mixture at constant temperature and pressure using vector concepts. [16]
7. The void volume of a sphere of porosity P and radius R is filled with a liquid containing a concentration C_o of a dissolved organic compound. The sphere is placed in a small volume v of the same liquid containing a concentration Y_o of the organic compound. As a result of the concentration difference, some of the organic compound is transferred from the inside of the sphere to the surrounding liquid. Within the sphere, diffusion controls the transport of the compound. The external liquid is mechanically mixed. Determine the rate at which the organic compound is transferred from the sphere to the surrounding liquid. [16]
8. Find the Laplace inverse transform following functions.

(a) $\frac{s}{s^2 - 2s + 5}$

(b) $\frac{k_p}{s(\tau_p s + 1)^2}$ Where k_p and τ_p are constants.

(c) $\frac{k_c}{(s+1)(5s+1)\left(\frac{s}{2}+1\right)}$ where k_c is constant. [4+6+6]

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1. Consider a gas containing an entrained mist of nonvolatile tar located inside the cylinder of a reciprocating compressor. Determine the work required to compress the gas adiabatically and reversibly from its present pressure of 0.33 atm to a pressure of 1.0atm. Given molecular wt of gas is 24 and specific heat at constant volume, is constant at $6.2Btu/(lbmole)^{\circ}F$. The tar is always present as a mist in the ratio 0.2 lb of tar pound of transfer gas. The specific heat of tar is $0.5Btu/(Lb)^{\circ}F$. The initial cylinder volume is $0.4ft^3$. The initial cylinder pressure is 0.33 atm. The final cylinder pressure is to be 1.0 atm. [16]
2. Form a linear differential equation with constant coefficients for consecutive reversible reactions at constant volume. [16]
3. If the three thermodynamic variables P, V, and T are connected by a relation $(P,V,T) = 0$ show that :

$$\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$$
 [16]
4. (a) Find the maximum and minimum distances from origin to the curve
 $5x^2 + 6xy + 5y^2 - 8 = 0$
 (b) The temperature T at any point (x,y,z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ [8+8]
5. (a) Explain in detail the concept of unit vector.
 (b) Find a unit vector parallel to the sum of the vectors $R_1 = 2i + 4j - 5k$ and $R_2 = i + 2j + 3k$. [6+10]
6. Find the work done in moving a particle in the force field $F = 3x^2.i + (2xz.y).j + z.k$ along
 (a) The straight line from (0,0,0) to (2,1,3).
 (b) The curve define by $x^2 = 4y, 3x^3 = 8z$ from x= 0 to x=2. Use the concept of line integral [6+10]
7. A solid cylinder of infinite length and radius R has an initial temperature distribution $T = f(r)$, where r is the local radius. The surface at $r=R$ is suddenly brought to and maintained at the temperature T_1 . Determine the interior temperature as a function of time. Compute the temperature distribution if $T = f(r) = T_o = 70^{\circ}F$, $T_1 = 1500^{\circ}F$, $R = 2$ ft, and the solid is steel. [16]

8. Show that laplace transform of

(a) $\cos \omega t = \frac{s}{s^2 + \omega^2}$

(b) $e^{-at} \sin \omega t = \frac{\omega}{(s+a)^2 + \omega^2}$

(c) $t^n = \frac{n!}{s^{n+1}}$

[6+6+4]

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2. (a) Discuss the general problem that arises constantly in chemical Engineering.
(b) The fundamental laws governing each of the unit operations in chemical engineering and most readily expressed in the form of simple rate equations. Discuss. [8+8]
3. Find the temperature $u(x, t)$ in a bar which is perfectly insulated laterally, whose ends are kept at temperatures $0^\circ C$ and whose initial temperature in $^\circ C$ is $f(x) = x(10 - x)$ given that its length is 10 cm, constant cross section of area "1" cm^2 , density $10.6gm/cm^3$ thermal conductivity, $1.04Cal/cm^\circ CSec$ and specific heat $0.056Cal/gm^\circ C$. [16]
4. (a) Find the maximum and minimum distances from origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$
(b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$ [8+8]
5. Given $A = i - 2j - 3k$ $B = 2i + j - 3k$ $C = i + j + k$ Verify
(a) $A \cdot (B \times C) = B \cdot (C \times A)$.
(b) $(A \times B) \times C = (A \cdot C)B - (B \cdot C)A$. [8+8]
6. Derive the diffusion equation for a binary perfect gas mixture at constant temperature and pressure in terms of the mass linear velocity using vector concepts. [16]
7. Determine the equation relating the steady-state temperature of position in an isotropic rectangular parallelepiped are $0 \leq x \leq L$, $0 \leq y \leq D$, $0 \leq z \leq H$. Five sides are maintained at the temperature T_0 , and the remaining face (corresponding to $z = H$) is maintained at the temperature T_1 . [16]
8. Find the Laplace inverse transform following functions.
(a) $\frac{s}{s^2 - 2s + 5}$

(b) $\frac{k_p}{s(\tau_p s + 1)^2}$ Where k_p and τ_p are constants.

(c) $\frac{k_c}{(s+1)(5s+1)\left(\frac{s}{2}+1\right)}$ where k_c is constant. [4+6+6]

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1. N_0 grams of a solid material was placed in W g of water at time t_0 . The liquid was continuously stirred and maintained at a constant temperature. At the end of t_1 sec, N_1 g of solid remained undissolved. At the end of a very long period of time, N_2 g of solid remained undissolved. Set up the differential equation required to determine the rate of solution of the solid in terms of N_0 , N_1 , N_2 and t_0 , t_1 . Do not integrate the expression. [16]
2. Under certain conditions cane sugar in water is converted into dextrose at a rate which is proportional to the amount unconverted at any time. Of the 75 gm. at time $t = 0$, 8 gm. are converted during the first 30 minutes, find the amount converted in $1\frac{1}{2}$ hours. [16]
3. (a) If x increases at the rate of 2cm/Sec at the instant when $x = 3$ cm and $Y = 1$ cm at what rate must y be changing in order that the function $2xy - 3x^2y$ shall neither be increasing nor decreasing.
(b) Find the total differential coefficient of x^2y with respect to x when x and y are connected by the relation $x^2 + xy + y^2 = 1$. [8+8]
4. (a) Discuss in detail the usefulness of partial differential equations in solving chemical engineering problems with a specific problem and with its solution.
(b) Find du/dx if $f(x, y) = 3x^3y^2 + x \cos y$
where $\frac{\partial f}{\partial x} = 9x^2 + \cos y$
 $\frac{\partial f}{\partial y} = 6x^2 - x \sin y$ [6+10]
5. (a) Explain the vector or cross product. Show that the commutative law does not hold for vector product.
(b) Given $A = 2i + 2j - k$, $B = 6i - 3j + 2k$ find $A \times B$ [8+8]
6. (a) Explain the concept of line integral. Define circulation of a vector.
(b) Evaluate the line integral $\int_c F dr$ where $F = 3xyi + y^2j$ and the space curve c is the Curve in the xy - plane $y = 2x^2$ from $(0, 0)$ to $(1, 2)$ [6+10]
7. A hollow cylinder has the inner face ($r = R_0$) maintained at $T = f_0(\theta)$ and the outer face ($r = R_1$) maintained $T = f_1(\theta)$. Determine the steady-state temperature distribution within the cylinder. θ denotes a coordinate in a cylindrical coordinate system. [16]

8. A liquid feed stream of heat capacity C_p is passed through a small bore cylindrical pipe of radius a m at the rate of R kg/h. The pipe wall is maintained at a temperature of $T_1^\circ C$ and the entering liquid at $T_0^\circ C$. Determine the temperature of the liquid as a function of time and distance from the inlet if the pipe is initially full of hot liquid at $T_1^\circ C$ when the flow is suddenly started. The heat transfer coefficient is a function of distance x m along the pipe from the inlet and is expressed as $U = kx^{-1/2}$ where k is constant. Conduction within the liquid can be neglected and radial variation of temperature can be assumed to be zero. [16]
