Code No: RR322204

Set No. 1

III B.Tech II Semester Supplementary Examinations, November/December 2005
DIGITAL AND OPTIMAL CONTROL SYSTEMS
(Instrumentation & Control Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. Convolve the two signals given by $f(k) = \begin{cases} 0 & 0 \leq k \leq 3 \quad and \quad k \geq 9 \\ 2 & 4 \leq k \leq 8 \end{cases}$

$$f(k) = \begin{cases} 0 & 0 \leq k \leq 1 \quad and \quad k \geq 6 \\ 2 & 3 \leq k \leq 6 \end{cases}$$

   Illustrate the steps involved in the convolution [16]

2. Derive the state space model of discrete control system using direct programming method and draw its block diagram. [16]

3. Consider the following continuous control system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 4(t)$$

   y(t)=x_1(t)

   The control signal u(k) is now generated by processing the signal u(t) through a sampler and zero order hold. Study the controllability and observability properties of the system under this condition. Determine the values of the sampling period for which the discretised system may exhibit hidden oscillation. [16]

4. A sampled – data controlled system is shown below Show that the output of the system at sampling instants is zeroT=$\frac{2\pi n}{Ws}$,n positive integer figure1

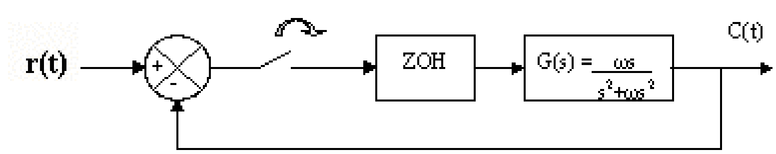

Figure 1:

5. Control a system, defined by

$$\dot{X} = AX + Bu \qquad Y = CX$$

   Where,

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

It is desired to have eigenvalues at -3.0 and -5.0 by using a state feedback control u = - KX. Determine the necessary feedback gain matrix k and the control signal u. [16]

6. (a) State and explain the minimum - time problems. Describe its performance index. [6]

   (b) Let $f(\mathrm{X})$ = -x_1x_2 and let g(X) =$x_1^2 + x_2^2 - 1$. What are the potential candidates for minima of f subject to the constraint g=0?. Show that the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$actually provide the minima. [10]

7. (a) Explain the concepts of variational calculus.

   (b) Explain the formulation of Variational Calculus using Himiltonian method. [8+8]

8. (a) Derive the relations required for obtaining a controllable realization algorithm of a given transfer matrix T(s).

   (b) Given $A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; B = $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$; C = $\begin{bmatrix} -2 & 1 & 2 \\ -2 & 2 & 0 \end{bmatrix}$; E = $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

   Find the corresponding transfer matrix. [8+8]

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1. Obtain the discrete-time output sequence c(k) of the shown below the input is a unit step sequence. Also, obtain the continuous-time output C(s) as shown in the figure 1

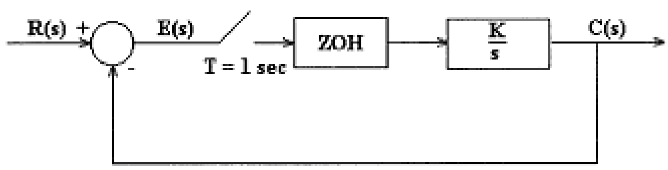

Figure 1:

2. Consider the discrete control system represented by the transfer function
$$G(z) = \frac{z^{-1}\left(1+z^{-1}\right)}{(1+0.5z^{-1})(1-0.5z^{-1})}$$
Obtain the state space representation in the diagonal form. [16]

3. Determine the stability of the following characteristic equations by using suitable tests.
   (a) $5z^2$- 2z + 2 = 0 [5]
   (b) z^3 - 0.2 z^2 - 0.25z + 0.05 = 0 [5]
   (c) z^4 - 1.7z^3 + 1.04z^2 - 0.268 z + 0.024 = 0 [6]

4. (a) Explain the design of the digital PID controller and PI controller in the Z-plane [10]
   (b) What are the advantages of digital PID controller over digital PI or PD controller. [6]

5. A discrete time regulator system has the plant equation
$$X(k+1) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} X(k) + \begin{bmatrix} 4 \\ 3 \end{bmatrix} u(k)$$
$$y(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} X(k) + 7u(k)$$
Design a state feedback control algorithm with u(k) = -KX(k) which places the closed loop characteristic roots at ±j0.5. [16]

6. (a) Explain the steps involved in solving an optimal control problem. [4]
   (b) Find the trajectories in the (t, x) plane which will extremize
   $$J(\mathrm{x}) = \int_0^{t_1} (t\,\dot{x} + \dot{x}^2)dt$$
   in each of the following two cases:
      i. t_1=1, x(0) = 1, x(1)=5 [6]
      ii. t_1=1, x(0) = 1, x(1) is free. [6]

7. (a) Explain the concepts of variational calculus.
   (b) Explain the formulation of Variational Calculus using Himiltonian method. [8+8]

8. (a) Explain the minimal D.O. realization algorithm

(b) The state space triple for a multivariable control system is given by

$$A = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & -2 & 0 & -4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix};\ \mathrm{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix};$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 0 \end{bmatrix}$$

Find the corresponding transfer matrix without calculating inverse of any matrix of order more than 2. [8+8]

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1. Given the transfer function
   $G(z) = \frac{4(z-1)(z^2+1.2z+1)}{(z+0.1)(z^2-0.3z+0.8)}$, Obtain
   (a) a series realization diagram and [8]
   (b) a parallel realization diagram, using pure delay elements z^{-1}. [8]

2. The block diagram of a sampled data system is shown below figure 1 , obtain the discrete time state model of the system. [16]

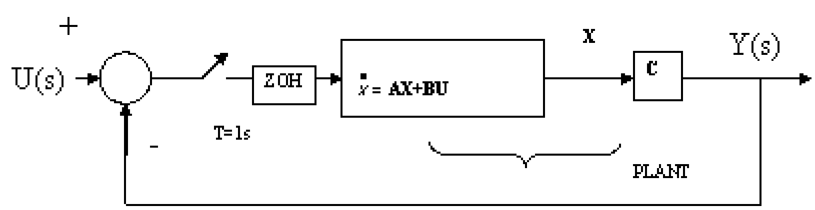

Figure 1:

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

3. Explain the Liapunov stability analysis of linear time invariant continuous time system and linear time invariant discrete time system. [16]

4. Show that the transfer function U(s) / E(s) of the PID controller shown below.
$$\frac{U(s)}{E(s)} = K_o \frac{T_1 + T_2}{T_1} \left[1 + \frac{1}{(T_1 + T_2)} + \left(\frac{T_1 T_2 \cdot s}{T_1 + T_2} \right) \right]$$
   Assume the gain **k** is very large compared with unity, or **k** >>**1**as shown in the figure 2

5. Consider an n^{th} order, single input system X(k+1) = AX(k)+bu(k) and use a feedback of the form u(k)=-KX(k)+r(k) where r is the reference input signal. Show that the zeros of the system are invariant under state feedback. [16]

6. (a) State and explain the minimum - time problems. Describe its performance index. [6]

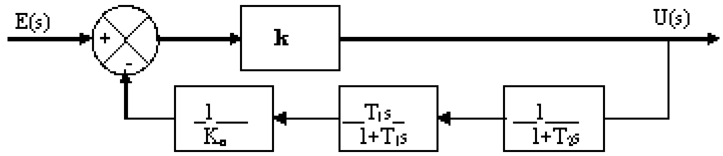

Figure 2:

(b) Let f(X) = -x_1x_2 and let g(X) =$x_1^2 + x_2^2 - 1$. What are the potential candidates for minima of f subject to the constraint g=0?. Show that the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$actually provide the minima. [10]

7. (a) With suitable diagrams illustrate the one point is fixed end, terminal time t_1 free and x(t_1) is specified problem and derive the necessary conditions of variational calculus.

   (b) For the system
   $\frac{d^2y}{dt^2} = u$
   with $|u| \leq 1$, find the control which drives the system from an arbitrary initial state to the origin in a condition satisfying $|y| \leq 0.5$ in the minimum time. [8+8]

8. (a) Derive the transfer matrix relation from state space representation. [6]

   (b) The state space triple (A, B, C) of a system is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{bmatrix}; \text{ B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; \text{ C} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   Calculate the input and output decoupling zeros, if any. Is the matrix A cyclic? Find out the transfer matrix T(s). [10]

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1. Given the discrete-time system
   $G(z) = \frac{2+2.2z^{-1}+0.2z^{-2}}{1+0.4z^{-1}-0.12z^{-2}}$
   Realize the system in
   (a) the parallel scheme and [8]
   (b) the ladder scheme using pure delay elements z^1. [8]

2. A second order multivariable system is described by the following equation

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 1/2 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ u_3(k) \end{bmatrix}$$

   Convert the state variable model into a transfer function matrix. [16]

3. (a) Discuss the Liapunov stability analysis for Linear Time Invariant (LTI) discrete time system. [6]
   (b) Determine the stability of the origin of the system given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.5 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

   [10]

4. Show that the transfer function U(s) / E(s) of the PID controller shown below.

$$\frac{U(s)}{E(s)} = K_o \frac{T_1 + T_2}{T_1} \left[1 + \frac{1}{(T_1 + T_2)} + \left(\frac{T_1 T_2 \cdot s}{T_1 + T_2}\right)\right]$$

   Assume the gain **k** is very large compared with unity, or **k** >>**1**as shown in the figure 1

5. (a) State the necessary and sufficient conditions for the arbitrary pole placement. [6]
   (b) Explain Ackerman's formula used for the determination of the state feedback gain matrix K. [10]

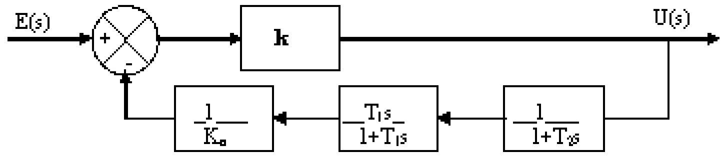

Figure 1:

6. (a) State and explain the minimum - time problems. Describe its performance index. [6]

   (b) Let f(X) = -x_1x_2 and let g(X) $=x_1^2 + x_2^2 - 1$. What are the potential candidates for minima of f subject to the constraint g=0?. Show that the points $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$actually provide the minima. [10]

7. (a) Explain the summary of the procedure for solving optimal control problems using Hamiltonian Formulation of Variational Calculus.

   (b) Find the curve with the minimum arc length joining the point (0, 0) and the line θ(t) = 2 − t. [8+8]

8. (a) Derive the transfer matrix relation from state space representation. [6]

   (b) The state space triple (A, B, C) of a system is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{bmatrix}; \ \mathrm{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; \ \mathrm{C} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   Calculate the input and output decoupling zeros, if any. Is the matrix A cyclic? Find out the transfer matrix T(s). [10]

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