Code No: RR210403 **Set No. 1**

II B.Tech I Semester Supplementary Examinations, November 2006
PROBABILITY THEORY & STOCHASTIC PROCESS
(Common to Electronics & Communication Engineering and Electronics & Telematics)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. A binary communication channel carries data as one of the two types of signals denoted by 0 and 1. Owing to noise a transmitted 0 is sometimes received as 1 and a transmitted 1 is sometimes received as a 0. For a given channel, assume a probability of 0.94 that a transmitted 0 is correctly received as a 0 and a probability of 0.91 that a transmitted 1 is received as a 1.Further assume a probability of 0.45 of transmitting a 0. If a signal is sent, Determine
   (a) probability that a 1 is received.
   (b) Probability that a 0 was received
   (c) Probability that a 1 was transmitted, given that a 1 was received
   (d) Probability that a 0 was transmitted, given that a 0 was received
   (e) Probability of as error

[3+3+4+4+2]

2. Two discrete random variables X and Y have joint p.m.f. given by the following table

X ↓	1	2	3	Y ←
1	1/12	1/6	1/12	
2	1/6	1/4	1/12	
3	1/12	1/12	0	

   Compute the probability of each of the following events
   (a) X ≤ 1½
   (b) XY is even
   (c) Y is even given that X is even.

[5+5+6]

3. (a) Let x be the random Variable with probability law $P(X = r) = q^{r-1}p, r = 1, 2, 3.....$ Find the moment generating function & hence mean & Variance, Assume p+q=1
   (b) The random Variable X has characteristic function is given by
   $\phi'(t) = 1 - |t|, |t| <= 1$

= 0,|t|>1
Find the density function of random variable X

[9+7]

4. A class of modulation signal is modulated by

$$X_c(t) = AX(t)Cos(\varpi_c t+\theta)$$

Where x(t) is the message signal and A Cos $(\varpi_c t + \theta)$ is the carrier. The message Signal x(t) is modeled as a zero mean stationary random process with the autocorrelation function $R_{xx}(\tau)$ and the PSD $G_x(f)$. The carrier amplitude A and frequency ϖ_c are assumed to be constants and the initial carrier phase θ is assumed to be a random variable uniformly distributed in the interval $(-\Pi, \Pi)$. Further more x(t) and θ are assumed to be independent.

   (a) Show that $X_c(t)$ is stationary
   (b) Find the PSD of $X_c(t)$.

[8+8]

5. (a) Find the PSD of a random process $z(t) = X(t) + y(t)$ where x(t) and y(t) are zero mean, individual random process.
   (b) A wss random process x(t) is applied to the input of an LTI system whose impulse response is $5t.e^{-2t}$ The mean of x(t) is 3. Find the output of the system.

[8+8]

6. (a) Explain the external noise sources of random noise.
   (b) Calculate the rms noise voltage generated in a bandwidth if 15 kHz by a resistor of 2kΩ operating at 20^0c. Find the noise power over this bandwidth. Find the noise PSD.

[8+8]

7. (a) Discuss the significance of noise equivalent temperature of an electronic system.
   (b) Evaluate the equivalent noise temperature of a two port device with a matched source and a matched load.

[8+8]

8. (a) Obtain the Shannon - Hartley law giving the, relation amongst channel capacity, bandwidth and signal to noise ratio of a continuous system.
   (b) Consider a message sequence having alphabets Q_1, Q_2, Q_3 and Q_4 with probabilities 1/2, 1/4, 1/8 and 1/8 respectively.
      i. Calculate the entropy of the message sequence,
      ii. Find the information rate of the message rate is 1 message / second,

iii. What is the rate at which binary signals are transmitted if the signal is sent after encoding Q_1, Q_2, Q_3 and Q_4 is 00, 01, 10 and 11.

[8+2+3+3]

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Code No: RR210403 **Set No. 2**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) State and prove Bayes theorem of probability.
   (b) In a single throw of two dice, what is the probability of obtaining a sum of at least 10?

[8+8]

2. (a) Define the joint distribution function. Explain how marginal density functions are Computed given their joint distribution functions.
   (b) A product is classified according to the number of defects it contains (X1) and the factory that produces it (X2). The joint probability distribution is given by:

X2 →	1	2
X1 ↓		
0	1/8	1/16
1	1/16	1/16
2	3/16	1/8
3	1/8	

   (a) Find the marginal distribution of X1
   (b) Find the conditional distribution of X1 when X2 is equal to 1
   (c) Are the variables X1 and X2 independent?

[7+9]

3. (a) If X is random variable, show that var(aX+b) = a^2 var(X)
   (b) A random variable z is uniformly distributed having Probability density function
   $f_Z(z) = 1/2, \quad -1 <= Z <= 1$
   $= 0$, otherwise
   Show that the random variables X=Z and Y=Z^2 are un-correlated despite of the fact that they are statistically dependant.

[6+10]

4. (a) Explain the concept of Random process.

(b) An ergoic random process is known to have an auto correlation function of the form

$$R_{xx}(\tau) = 1 - |\tau|, |\tau| \leq 1$$

$$= 0, |\tau| > 1$$

Show the spectral density is given by

$$S_{xx}(\omega) = \left[\frac{\text{Sin}\omega/2}{\omega/2}\right]^2$$

[8+8]

5. A Random process n(t) has a power spectral density g(f) = $\eta/2$ for $\alpha \leq f \leq \alpha$ Random process is passed through a low pass filter which has transfer function H(f)=2 for $-f_m \leq f \leq f_m$ and H(f)= 0 otherwise. Find the PSD of the waveform at the o/p of the filter. [16]

6. (a) What are the sources of flicker noise and how can it be reduced?
   (b) How noise equivalent bandwidth of a electronic circuit can be estimated?

[8+8]

7. (a) Bring out the difference between narrowband and broadband noises
   (b) Describe the quadrature representation of narrowband noise.

[8+8]

8. (a) Define the terms: Information, Self Information and Information rate with respect to a source.
   (b) A source generates eight symbols: A, B, C, D, E, F, G and H with probabilities 0.5, 0.2, 0.1, 0.05, 0.05, 0.05, 0.03 and 0.02. Find the entropy and information rate if the symbols are generated at a rate of 1000 / Sec.

[6+10]

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Code No: RR210403

Set No. 3

II B.Tech I Semester Supplementary Examinations, November 2006
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) State and prove Bayes theorem of probability.
   (b) In a single throw of two dice, what is the probability of obtaining a sum of at least 10?

[8+8]

2. The Rayleigh density function is given by

$$f(x) = x\,e^{-x^2/2} \quad x \geq 0$$
$$= 0\, x < 0$$

   (a) Prove that f (x) satisfies the properties of the p.d.f.
      i. f(x)$\geq$ 0 for all x and
      ii. $\int_{\infty}^{\infty} f(x)\,dx = 1$
   (b) Find the distribution function F (x)
   (c) Find P(0.5 < x $\leq$ 2)
   (d) Find P(0.5 $\leq$ x < 2).

[2+2+4+4+4]

3. (a) State and Prove any four properties of characteristic function.
   (b) Find the density function of the distribution for which the characteristic function is

$$\emptyset(t) = e^{-\frac{-t^2\sigma^2}{2}}$$

[10+6]

4. (a) If the auto correlation function of a wss process is R(τ) = k .$e^{-k(\tau)}$, show that its spectral density is given by $S(\omega) = \frac{2}{1+\left(\frac{\omega}{k}\right)^2}$
   (b) Find the PSD of a random process x(t) if E[x(t)] = 1 and $R_{xx}(\tau) = 1 + e^{-\alpha|\tau|}$

[8+8]

5. A Random process n(t) has a power spectral density g(f) = $\eta/2$ for $\alpha \leq f \leq \alpha$ Random process is passed through a low pass filter which has transfer function H(f)=2 for $-f_m \leq f \leq f_m$ and H(f)= 0 otherwise. Find the PSD of the waveform at the o/p of the filter. [16]

6. (a) What are the characteristics of shot noise?
   (b) What are the important requirements of the front-end stage of a communication receiver in the point of view of noise?

   [8+8]

7. (a) What are the precautions to be taken in cascading stages of a network in the point of view of noise reduction?
   (b) What is the need for band limiting the signal towards the direction increasing SNR.

   [8+8]

8. (a) A code is composed of dots and dashes. Assume that a dash is three times as long as the dot and has one-third the probability of occurrence.
   Find,
      i. The information in a dot and that in a dash, and
      ii. The entropy in the dot - dash code.
   (b) Suppose 100 voltage levels are employed to transmit 100 equally likely messages. Assume the system to be a Gaussian channel with $\lambda = 3.5$ and bandwidth B = 104 Hz. Find S/N.

   [8+8]

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Code No: RR210403 **Set No. 4**

II B.Tech I Semester Supplementary Examinations, November 2006
PROBABILITY THEORY & STOCHASTIC PROCESS
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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) If A and B are any events, not necessarily mutually exclusive events, derive an expression for probability of A Union B. When A and B are mutually exclusive, what happens to the above expression derived?
   (b) Define the term Independent events. State the conditions for independence of
      i. any two events A and B.
      ii. any three events A, B and C.
   (c) A coin is tossed. If it turns up heads, two balls will be drawn from box A, otherwise, two balls will be drawn from box B. Box A contains three black and five white balls. Box B contains seven black and one white balls. In both cases, selections are to be made with replacement. What is the probability that Box A is used, given that both balls drawn are black?

[5+6+5]

2. The Rayleigh density function is given by

$$\begin{aligned} f(x) &= x\, e^{-x^2/2} \quad x \geq 0 \\ &= 0\, x < 0 \end{aligned}$$

   (a) Prove that f (x) satisfies the properties of the p.d.f.
      i. f(x)$\geq$ 0 for all x and
      ii. $\int_{\infty}^{\infty} f(x)\, dx = 1$
   (b) Find the distribution function F (x)
   (c) Find P$(0.5 < x \leq 2)$
   (d) Find P$(0.5 \leq x < 2)$.

[2+2+4+4+4]

3. (a) For the random variable ? X ? whose density function is

$$f(x) = \frac{1}{b-a}, \qquad a \leq x \leq b$$
$$0, \quad otherwise$$

Determine

   i. Moment generating function
   ii. Mean and Variance
   (b) Prove that E(X) = E(X/Y), where X and Y are two random variables

[8+8]

4. (a) Explain the concept of Random process.
   (b) An ergoic random process is known to have an auto correlation function of the form

$$R_{xx}(\tau) = 1 - |\tau|, |\tau| \leq 1$$
$$= 0, |\tau| > 1$$

Show the spectral density is given by

$$S_{xx}(\omega) = \left[\frac{\text{Sin}\omega/2}{\omega/2}\right]^2$$

[8+8]

5. A random process x(t) is applied to a network with impulse response

$$h(t) = u(t)texp(-bt)$$

where b>0 is a constant. The cross-correlation of x(t) with the o/p y(t) is known to have the same form.

$$R_{xx}(\tau) = u(\tau)\,\tau\exp(-b\tau)$$

   (a) Find the auto correlation of y(t)?
   (b) What is the average power in y(t)?

[8+8]

6. (a) What is shot noise? How is it qualified?
   (b) How the spectral density of White noise is denoted

[8+8]

7. (a) Discuss the significance of noise equivalent temperature of an electronic system.
   (b) Evaluate the equivalent noise temperature of a two port device with a matched source and a matched load.

[8+8]

8. (a) Calculate the average information content in English language, assuming that each of the 26 characters in alphabet occurs with equal probability.
   (b) Plot the channel capacity C versus B with S/N = constant for the Gaussian channel.
   (c) Explain the concept of "amount of information and redundancy".

[5+5+6]

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