Code No: R059210401 **Set No. 1**

II B.Tech I Semester Regular Examinations, November 2006
PROBABILITY THEORY & STOCHASTIC PROCESS
(Common to Electronics & Communication Engineering, Electronics & Telematics and Electronics & Computer Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

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1. (a) Define and explain the following with an example:
   i. Equally likely events
   ii. Exhaustive events
   iii. Mutually exclusive events

   (b) Give the classical definition of probability.

   (c) Find the probability of three half-rupee coins falling all heads up when tossed simultaneously. [6+4+6]

2. (a) Define conditional distribution and density functions and explain their properties.

   (b) A continuous random variable X has a P.D.F f(x) = $3X^2$, 0 ⇐ x ⇐ 1. Find 'a' and 'b' such that
   i. P{x ⇐ a} = P{x > a} and
   ii. P{x > b} = 0.05.. [10+6]

3. (a) State and prove properties of variance of a random variable

   (b) Let X be a random variable defined by the density function
   $f_X(x) = \begin{cases} \frac{\pi}{16}\cos(\pi x/8) & -4 \le x \le 4 \\ 0 & elsewhere \end{cases}$ Find E[3X] and E[X^2]. [8+8]

4. (a) Define and explain joint distribution function and joint density function of two random variables X and Y.

   (b) If the function $f(x,y) = \begin{cases} be^{-2x}\cos(y/2) & 0 \le x \le 1 \ , \ \ 0 \ \le y \le \pi \\ 0 & elsewhere \end{cases}$, where 'b' is a positive constant, is valid joint probability density function, find 'b' [8+8]

5. (a) let Y = X_1 + X_2 + ...+X_N be the sum of N statistically independent random variables X_i, i=1,2.............. N. If Xi are identically distributed then find density of Y, f_y(y).

   (b) Consider random variables Y_1 and Y_2 related to arbitrary random variables X and Y by the coordinate rotation. Y_1=X Cos θ + Y Sin θ Y_2 = -X Sin θ + Y Cos θ
   i. Find the covariance of Y_1 and Y_2, C_{Y1Y2}
   ii. For what value of θ, the random variables Y_1 and Y_2 uncorrelated. [8+8]

6. Let X(t) be a stationary continuous random process that is differentiable. Denote its time derivative by $\dot{X}(t)$.

   (a) Show that $E[X(t)] = 0$.

   (b) Find $R_{\dot{X}\dot{X}}(\tau)$ in terms of $R_{\dot{X}\dot{X}}(\tau)$ [8+8]

7. (a) Determine which of the following functions are valid PSDS.

   i. $\frac{\omega^2}{\omega^6 3\omega^2+3}$

   ii. $\exp[-(\omega-1)^2]$

   iii. $\frac{\omega^2}{\omega^4+1} - 8(\omega)$

   iv. $\frac{\omega^4}{1+\omega^2+j\omega^6}$. [4 ×3 = 12]

   (b) Define RMS bandwidth of PSD and explain. [4]

8. (a) For the network shown in figure 1 find the transfer function of the system. [8]

   (b) Define the following systems

   i. LTI system

   ii. Causal system

   iii. stable system

   iv. Noise Bandwidth. [4×2=8]

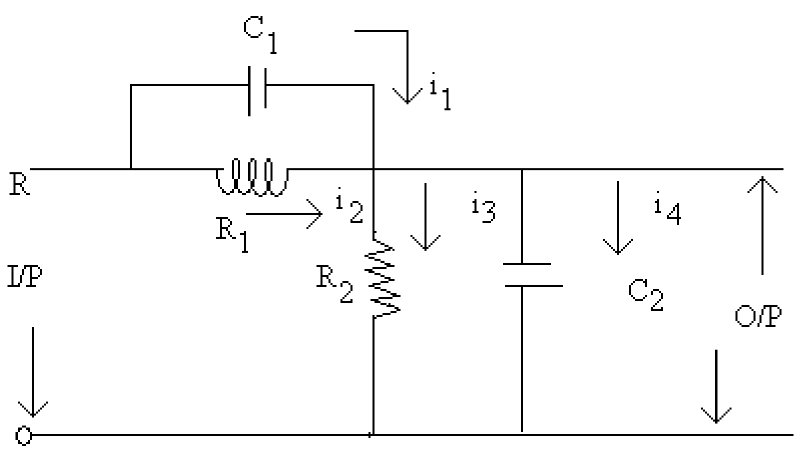

Figure 1:

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1. (a) What is ‘ total probability’ and show that total probability P(A) is $\mathrm{P(A) = P(A \cap S) = P[U^{N} n = 1(A \cap B_n)] =^{N} \sum_{n=1} P(A \cap B_n)}$ with Venn diagram.

   (b) In a box there are 100 resistors having resistance and tolerance as shown in table. Let a resistor be selected from the box and assume each resistor has the same likelihood of being chosen. For the three events; A as “draw a 47Ω resistor,” B as “draw a resistor with 5% tolerance” and C as “draw a 100 Ω resistor” calculate the joint probabilities.

   Table 1

   Numbers of resistors in a box having given resistance and tolerance.

Resistance(Ω)	Tolerance		
	5%	10%	Total
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

[10+6]

2. (a) Define Random variable and give the concept of random variable.

   (b) In an experiment of rolling a die and flipping a coin. The random variable(X) is choosen such that

      i. a coin head (H) outcome corresonds to positive values of X that are equal to the numbers that show upon the die and

      ii. a coin tail (T) outcome corresponds to negative values of X that are equal in magnitude to twice the number that shows on die. Map the elements of random variable X into points on the real line and explain.

   (c) In experiment where the pointer on a wheel of chance is spun. The possible outcomes are the numbers from 0 to 12 marked on the wheel. The sample space consists of the numbers in the set $\{0 < S <= 12\}$ and if the random variable X is defined as X = X(S) = S^2, map the elements of random variable on the real line and explain. [4+6+6]

3. (a) Find the expected value of the function g(X) = X^2, where X is a random variable defined by the density , $f_X(x) = ae^{-ax}u(x)$, where a is constant.

   (b) For the Rayleigh density function $f_X(x) = \begin{cases} \frac{2(x-a)}{b}e^{-(x-a)/b} & x > a \\ 0 & x < a \end{cases}$, show that $E[X] = a + \sqrt{\pi b/4}$ and $\sigma_X^2 = b(4-\pi)/4$. [6+10]

4. (a) Define and explain conditional probability mass function. Give its properties.

   (b) The joint probability density function of two random variables X and Y is given by $f(x, y) = \begin{cases} C(2x+y) & 0 \leq x \leq 1\ ,\ \ 0\ \leq y \leq 2 \\ 0 & elsewhere \end{cases}$. Find

      i. the value of 'C'

      ii. marginal distribution functions of X and Y. [8+8]

5. (a) let Y = X_1 + X_2 + ...+X_N be the sum of N statistically independent random variables X_i, i=1,2.............. N. If Xi are identically distributed then find density of Y, f_y(y).

   (b) Consider random variables Y_1 and Y_2 related to arbitrary random variables X and Y by the coordinate rotation. Y_1=X Cos θ + Y Sin θ Y_2 = -X Sin θ + Y Cos θ

      i. Find the covariance of Y_1 and Y_2, C_{Y1Y2}

      ii. For what value of θ, the random variables Y_1 and Y_2 uncorrelated. [8+8]

6. (a) State the properties of auto correlation function.

   (b) Given the auto correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 25 + 4/(1 + 6\tau^2)$ Find mean and variance of process X(t). [6+10]

7. (a) Consider a random process X(t) = cos (ωt + θ) where W is a real constant and θ is a uniform random variable in (0, π /2)). Show that $\chi(t)$ is not a WSS process. Also find the average power in the process

   (b) State and prove wiener-khinchin relation. [8+8]

8. (a) A Signal x(t) = u(t) exp (-αt) is applied to a network having an impulse response h(t)= ω u(t) exp (-ω t). Here α & ω are real positive constants. Find the network response? (6M)

   (b) Two systems have transfer functions $H_1(\ \omega)$ & $H_2(\ \omega)$. Show the transfer function H(ω) of the cascade of the two is H(ω) =$H_1(\ \omega)$ H_2 (ω).

   (c) For cascade of N systems with transfer functions $H_n(\omega)$, n=1,2,.... .N show that H(ω) = $\pi H_n(\omega)$. [6+6+4]

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   i. Equally likely events
   ii. Exhaustive events
   iii. Mutually exclusive events

   (b) Give the classical definition of probability.

   (c) Find the probability of three half-rupee coins falling all heads up when tossed simultaneously. [6+4+6]

2. (a) Define and give the concept of random variable.

   (b) Define conditional distribution and density functions and explain their properties. [6+10]

3. (a) State and prove properties of moment generating function of a random variable X

   (b) The characteristic function for a random variable X is given by $\Phi_X(\omega) = \frac{1}{(1-j2\omega)^{N/2}}$. Find mean and second moment of X. [8+8]

4. (a) Define and explain joint distribution function and joint density function of two random variables X and Y.

   (b) If the function $f(x,y) = \begin{cases} be^{-2x}\cos(y/2) & 0 \le x \le 1\ ,\ \ 0\ \le y \le \pi \\ 0 & elsewhere \end{cases}$, where 'b' is a positive constant, is valid joint probability density function, find 'b' [8+8]

5. For two random variables X and Y
   $\mathrm{f_{X,Y}(x,y) = 0.5\delta(x+1)\delta(y) + 0.1\delta(x)\delta(y) + 0.1\delta(x)\delta(y-2) + 0.4\,\delta(x-1)}$
   $\mathrm{\delta(y+2) + 0.2\delta(x-1)\delta(y-1) + 0.5\delta(x-1)\delta(y-3)}$

   Find

   (a) the correlation
   (b) the covariance and
   (c) the correlation Coefficient of X and Y
   (d) are X and Y either uncorrelated or orthogonal. [16]

6. (a) Define cross correlation function of two random processes X(t) and Y(t) and state the properties of cross correlation function.

(b) let two random processes X(t) and Y(t) be defined by
X(t) = A cos ω_0t + B sin ω_0t
Y(t) = B cos ω_0t - A sin ω_0t
Where A and B are random variables and ω_0 is a constant. Assume A and B are uncorrelated, zero mean random variables with same variance. Find the cross correlation function R_{XY} (t,t+τ) and show that X(t) and Y(t) are jointly wide sense stationary. [6+10]

7. (a) Consider a random process X(t) = cos (ωt + θ) where W is a real constant and θ is a uniform random variable in (0, π /2)). Show that $\chi(t)$ is not a WSS process. Also find the average power in the process
   (b) State and prove wiener-khinchin relation. [8+8]

8. (a) Define the following random processes
      i. Band Pass
      ii. Band limited
      iii. Narrow band. [3×2 = 6]
   (b) A Random process X(t) is applied to a network with impulse response h(t) = u(t) exp (-bt)
      where b > 0 is W constant. The Cross correlation of X(t) with the output Y (t) is known to have the same form:
      R_{XY} (Y) = u(Y)Y exp (-bY)
      i. Find the Auto correlation of Y(t)
      ii. What is the average power in Y(t). [6+4]

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Code No: R059210401 **Set No. 4**

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1. (a) What is Bayes' theorem? Explain.
   (b) Determine probabilities of system error and correct system transmission of symbols for an elementary binary communication system shown in figure 1 consisting of a transmitter that sends one of two possible symbols (a 1 or a 0) over a channel to a receiver. The channel occasionally causes errors to occur so that a '1' show up at the receiver as a '0', and vice versa. Assume the symbols '1' and '0' are selected for a transmission as 0.6 and 0.4 respectively. [6+10]

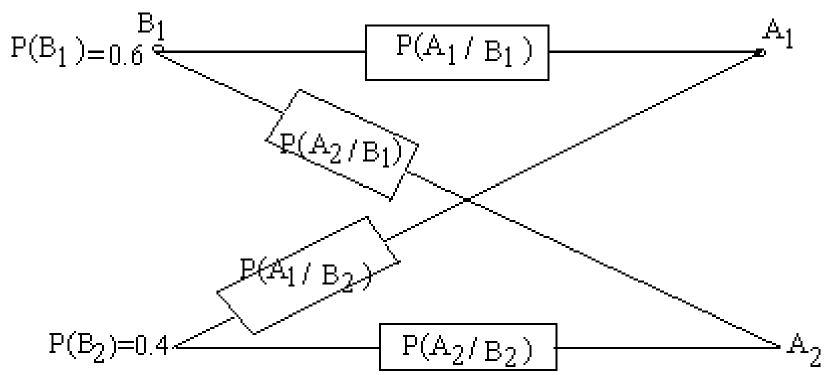

Figure 1:

2. (a) When a random variable X is said to be guassian. Exlain its importance
   (b) Define and explain the following density functions
      i. Binomial
      ii. Exponential
      iii. Uniform
      iv. Rayleigh. [4+12]

3. (a) The density function of a random variable X is $gX(x) = \begin{cases} 5e^{-5x} & 0 \leq x \leq \infty \\ 0 & elsewhere \end{cases}$
   Find
      i. E[X],

ii. $E[(X-1)^2]$

iii. E[3X-1]

(b) If the mean and variance of the binomial distribution are 6 and 1.5 respectively. Find $E[X - P(X \geq 3)]$.. [8+8]

4. (a) Define and explain the properties of conditional density functions.

   (b) Joint probabilities of two random variables X and Y are given in table 2

Y \ X	1	2	3
1	0.2	0.1	0.2
2	0.15	0.2	0.15

Figure 2:

Find out

i. joint and marginal distribution functions and plot.

ii. joint and marginal density functions and plot. [8+8]

5. (a) Random variables X and Y have the joint density function
   $f_{X,Y}(x,y) = (x+y)^2/40 \quad -1 < x < 1 \quad \text{and} \quad -3 < y < 3$

   find all the third order moments for X and Y.

   (b) random variables Z and W are defined by. Z= X+aY W= X-aY. Where a is a real number. Determine 'a' such that Z and W are orthogonal. [8+8]

6. (a) State the properties of auto correlation function.

   (b) Given the auto correlation function for a stationary ergodic process with no periodic components is $R_{XX}(\tau) = 25 + 4/(1+6\tau^2)$ Find mean and variance of process X(t). [6+10]

7. (a) Find the ACF of the following PSD's

   i. $S_{XX}(\omega) = \frac{157+12\omega^2}{(16+\omega^2)(9+\omega^2)}$

   ii. $S_{XX}(\omega) = \frac{8}{(9+\omega^2)^2}$

   (b) State and Prove wiener-Khinchin relations. [8+8]

8. (a) Determine which of the following impulse response do not correspond to a system that is stable or realizable or both and state why

i. h(t) = u(t+3)
ii. h(t) = u(t) e^{-t2}
iii. h(t) = e^{+} sin(ω_0T), ω_0 : real constant.
iv. h(t) = $u(t)e^{-3t}$, ω_0 : real constant

(b) A random process X(t) = A Sin (ω_0t $+\theta$) where A & $\omega 0$ are real positive constants & θ is a random variable uniformly distributed in the internal ($-\pi$,π) is applied to the network shown in figure 3. Find an expression for the networks response? [8+8]

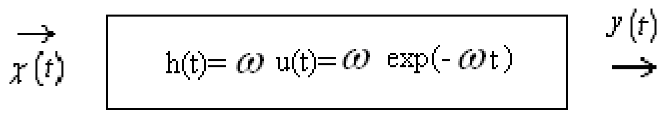

Figure 3:

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