

II B.Tech I Semester Supplementary Examinations, November 2006
PROBABILITY & RANDOM VARIABLES
 (Common to Electronics & Communication Engineering and Electronics & Telematics)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
All Questions carry equal marks

1. (a) If A and B are any events, not necessarily mutually exclusive events, derive an expression for probability of A Union B. When A and B are mutually exclusive, what happens to the above expression derived?
- (b) Define the term Independent events. State the conditions for independence of
 - i. any two events A and B.
 - ii. any three events A, B and C.
- (c) A coin is tossed. If it turns up heads, two balls will be drawn from box A, otherwise, two balls will be drawn from box B. Box A contains three black and five white balls. Box B contains seven black and one white balls. In both cases, selections are to be made with replacement. What is the probability that Box A is used, given that both balls drawn are black?

[5+6+5]

2. The Rayleigh density function is given by

$$f(x) = x e^{-x^2/2} \quad x \geq 0$$

$$= 0 \quad x < 0$$

- (a) Prove that f (x) satisfies the properties of the p.d.f.
 - i. $f(x) \geq 0$ for all x and
 - ii. $\int_{-\infty}^{\infty} f(x) dx = 1$
- (b) Find the distribution function F (x)
- (c) Find $P(0.5 < x \leq 2)$
- (d) Find $P(0.5 \leq x < 2)$.

[2+2+4+4+4]

3. (a) For the random variable ? X ? whose density function is

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

$$0, \text{ otherwise}$$

Determine

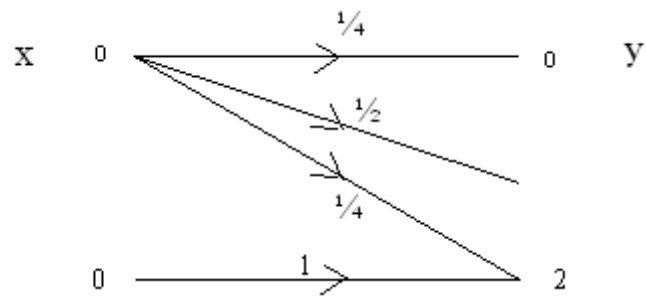
- i. Moment generating function
 - ii. Mean and Variance
- (b) Prove that $E(X) = E(X/Y)$, where X and Y are two random variables
[8+8]
4. (a) If the auto correlation function of a wss process is $R(\tau) = k \cdot e^{-k(\tau)}$, show that its spectral density is given by $S(\omega) = \frac{2}{1+(\frac{\omega}{k})^2}$
 (b) Find the PSD of a random process $x(t)$ if $E[x(t)] = 1$ and $R_{xx}(\tau) = 1 + e^{-\alpha|\tau|}$
[8+8]
5. (a) Find the PSD of a random process $z(t) = X(t) + y(t)$ where $x(t)$ and $y(t)$ are zero mean, individual random process.
 (b) A wss random process $x(t)$ is applied to the input of an LTI system whose impulse response is $5t \cdot e^{-2t}$. The mean of $x(t)$ is 3. Find the output of the system.
[8+8]
6. (a) What are the causes of thermal noise?
 (b) What are the causes of shot noise?
[8+8]
7. In TV receivers, the antenna is often mounted on a tall mast and a long lossy cable is used to connect the antenna and receiver. To overcome the effect of noisy cable, a preamplifier is mounted on the antenna. The parameters of the different stages are

Preamplifier gain	= 20 dB
Preamplifier Noise figure	= 6 dB
Lossy cable noisy figure	= 3 dB
Cable Loss	= -20 dB
Receiver front end gain	= 60 dB
Receiver Noise figure	= 16 dB

 Determine the overall noise figure of the system.
[16]
8. (a) The voice frequency-modulating signal of a PCM system is quantized in 16 levels with the following probabilities,

$P_1 = P_2 = P_3 = P_4 = 0.1$
$P_5 = P_6 = P_7 = P_8 = 0.05$
$P_9 = P_{10} = P_{11} = P_{12} = 0.075$
$P_{13} = P_{14} = P_{15} = P_{16} = 0.025$

 Find the information rate taking the band limiting frequency of the modulating signal at 3 KHz.
 (b) For the communication system where channel characteristic is given below figure8b,



Verify that $I(x, y) = I(y, x)$.

Figure 8b

[6+10]
