II B.Tech I Semester Regular Examinations, November 2006
MATHEMATICS-III
(Common to Electrical & Electronic Engineering, Mechanical Engineering, Chemical Engineering, Mechatronics, Metallurgy & Material Technology, Production Engineering, Aeronautical Engineering and Automobile Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Evaluate $4\int\limits_0^\infty \frac{x^2dx}{1+x^4}$ using $\beta - \Gamma$ functions
   (b) $\beta\left(m+\frac{1}{2}, m+\frac{1}{2}\right) = \frac{\prod}{m, 2^{4m-1}}\ \frac{1}{\beta(\text{m,m})}$
   (c) Evaluate $\int\limits_0^2 (8-x^3)^{1/3}dx$ using$\beta - \Gamma$ functions [5+5+6]

2. (a) Prove that $P_n(0)$=0 for n odd and $P_n\left(0\right) = \frac{(-1)^{\frac{n}{2}} n!}{2^n\left(\frac{n}{2}!\right)^2}$ if n is even.
   (b) Prove that J_2-J_0= 2 J_0” [8+8]

3. (a) If f(z) = u + iv is an analytic function and u-v = $\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^y-e^{-y}}$, find f(z) subject to the condition $f(\pi/2) = 0$
   (b) Separate the real and imaginary parts of log sin z [8+8]

4. (a) Evaluate $\int\limits_c \frac{ze^z\,dz}{(z+2)^3}$ where c is $|z| = 3$ using Cauchy’s integral formula.
   (b) Evaluate $\int\limits_c (x^2+ixy)$dz from A(1,1) to B(2,8) along x=t y=t^3
   (c) Evaluate $\int\limits_C \left[\frac{e^z}{z^3} + \frac{z^4}{(z+i)^2}\right]$ dz where c: $|\,z\,| = 2$ Using Caucy’s integral theorem [5+5+6]

5. (a) State and prove Taylor’s theorem.
   (b) Find the Laurent series expansion of the function $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region 3< |z+2| <5. [8+8]

6. (a) Find the poles and the residues at each pole of $f\left(z\right) = \frac{\sin^2 z}{(z-\pi/6)^2}$
   (b) Find The poles and the residues at each pole of $f\left(z\right) = \frac{ze^z}{(z-1)^3}$. [5+5+6]
   (c) Evaluate $\int\limits_C \frac{\cos\pi z^2 dz}{(z-1)(z-2)}$ where c is $|\,z\,| = 3/2$. [5+5+6]

7. (a) Show that $\int\limits_0^{2\pi} \frac{d\theta}{a+bSin\theta} = \int\limits_0^{2\pi} \frac{d\theta}{a+bSin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, a > b> 0 using residue theorem.

(b) Evaluate by contour integration $\int_{0}^{\infty} \frac{dx}{1+x^2}$ [8+8]

8. (a) Find and plot the image of triangular region with vertices at (0,0), (1,0) (0,1) under the transformation w=(1-i) z+3.

   (b) If $w = \frac{1+iz}{1-iz}$find the image of |z| <1. [8+8]

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Code No: R059210201

Set No. 2

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Evaluate $\int\limits_0^{\pi/2} \sin^2\theta\cos^4\theta d\theta = \frac{\pi}{32}$ using $\beta - \Gamma$functions.
   (b) Prove that $\int\limits_0^{\infty} \sqrt{x}e^{-x^2}dx = 2\int\limits_0^{\infty} x^2e^{-x^4}dx$ using B-T functions and evaluate
   (c) Show that $\int\limits_0^{\infty} \frac{x^{m-1}}{(x+a)^{m+n}}dx = a^{-n}\beta(m,n)$ [5+6+5]
2. (a) Prove that J_0^2 +2 (J_1^2 + J_2^2+ ....) = 1.
   (b) Prove that x^4 = $\frac{8}{35}p_4(x) + \frac{4}{7}P_2(x) + \frac{1}{5}P_0(x)$. [8+8]
3. (a) If f(z) = u + iv is an analytic function and u-v = $\frac{\cos x+\sin x-e^{-y}}{2\cos x-e^y-e^{-y}}$, find f(z) subject to the condition $f(\pi/2) = 0$
   (b) Separate the real and imaginary parts of log sin z [8+8]
4. (a) Evaluate $\int\limits_c \frac{dz}{z^2e^z}$ where C is $|z| = 1$
   (b) Evaluate using Cauchy's integral formula $\int\limits_0^{1+i} z^2\ dz$ along y = x^2
   (c) Prove that $\int\limits_C \frac{dz}{(z-a)}$ $= 2\pi i$ where C is given by the equation 1z-a1=r [6+5+5]
5. (a) Expand as a Taylor series in f(z) =$\frac{2z^3+1}{z^2+1}$ about z=1
   (b) Express f(z) = $\frac{z}{(z-1)\,(z-3)}$ in a series of positive and negative powers Of (z-1) [8+8]
6. (a) Find the poles and residues at each pole $\frac{\cot z\ \cot\ \ hz}{z^3}$
   (b) Evaluate $\int\limits_C \frac{3\sin z.dz}{(z^2-\frac{\pi^2}{4})}$ where C is |z| = Π by residue theorem. [8+8]
7. (a) Evaluate by residue theorem$\int\limits_0^{2\pi} \frac{d\theta}{2+Cos\theta}$
   (b) Use the method of contour integration to evaluate $\int\limits_{-\infty}^{\infty} \frac{x^2dx}{(x^2+a^2)^3}$ [8+8]

8. (a) Show that the transformation w=z+1/z converts the straight line arg z=a(|a| $<$ $\pi/2$) in to a branch of the hyperabola of eccentricity sec a

   (b) Find the bilinear transformation which maps the points (0, 1, ∞) into the points (-1, -2, -i). [8+8]

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1. (a) Evaluate $\int\limits_0^1 \frac{x^2 dx}{\sqrt{1-x^5}}$ interms of β function.
   (b) Prove that $\int\limits_0^1 (1-x^n)^{1/n} dx = \frac{1}{n}\frac{\left[\Gamma\left(\frac{1}{n}\right)\right]^2}{2\Gamma(2/n)}$
   (c) Prove that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right)$ ....$\Gamma\left(\frac{n-1}{n}\right) = \frac{(2\prod)^{\frac{n-1}{2}}}{n^{1/2}}$ [5+5+6]
2. (a) Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0$(x) + P_1(x) t + P_2(x) t^2+ ....
   (b) Write $J_{5/2}$(x) in finite form. [8+8]
3. (a) Determine the analytic function w = u+iv where u = $\frac{2\cos x\ \cosh y}{\cos 2x\ +\cosh 2y}$ given that f(0) =1.
   (b) If cosec ($\pi/4$ + i α) = u + iv prove that $(u^2+v^2) = 2(u^2-v^2)$. [8+8]
4. (a)
   (b) Evaluate $\int\limits_0^{1+i} (x-y+ix^2) dz$ along
      i. z=0 to 1+I
      ii. The real axis from z=0 to 1 and their along a line parallel to the ‘ imaginary axis from z=1 to 1+i

   [8+8]
5. (a) Expand cos h z abouts z = πi
   (b) Find the Laurent series expansion of the function $\frac{z^2-1}{(z+2)\,(z+3)}$ if 2 < $|z|$ < 3
   [8+8]
6. (a) Find the poles $\frac{e^{iz}}{(z^2+1)}$ and corresponding residues.
   (b) Evaluate $\int\limits_c \frac{z}{(z-1)(z-2)^2}$ dz Where **c** is the circle | Z – 2 | = $\frac{1}{2}$ using residue theorm. [8+8]
7. (a) Evaluate by residue theorem $\int\limits_0^{2\pi} \frac{d\theta}{2+Cos\theta}$

(b) Use the method of contour integration to evaluate $\int\limits_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)^3}$ [8+8]

8. (a) Find the image of the domain in the z-plane to the left of the line x=-3 under the transformation $w=z^2$

   (b) Find the bilinear transformation which transforms the points z=2,1,0 into w=1,0,i respectively. [8+8]

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Code No: R059210201 **Set No. 4**

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Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
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1. (a) Show that $\int\limits_0^\infty x^m e^{-ax^n} dx = \frac{1}{na^{\frac{m+1}{n}}}\Gamma((m+1)/n)$ where n and m are positive constants.
   (b) Prove that $\int\limits_o^\infty e^{-y^{1/m}} dy = m\Gamma m$
   (c) Show that $\int\limits_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1}\beta(m+1, n+1)$ [5+5+6]

2. (a) Prove that $\frac{1}{\sqrt{1-2tx+t^2}} = P_0$(x) + P_1(x) t + P_2(x) t^2+ ....
   (b) Write $J_{5/2}$(x) in finite form. [8+8]

3. (a) Define analyticity of a complex function at a point P and in a domain D. Prove that the real and imaginary parts of an analytic function satisfy Cauchy Riemann Equations.
   (b) Show that the function defined by $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ at z $\neq$ 0 and f(0) = 0 is continuous and satisfies C-R equations at the origin but f$'$(0) does not exist. [8+8]

4. (a) Evaluate $\int\limits_c \frac{ze^z\, dz}{(z+2)^3}$ where c is $|z| = 3$ using Cauchy's integral formula.
   (b) Evaluate $\int\limits_c (x^2 + ixy)$dz from A(1,1) to B(2,8) along x=t y=t^3
   (c) Evaluate $\int\limits_C \left[\frac{e^z}{z^3} + \frac{z^4}{(z+i)^2}\right]$ dz where c: $|\,z\,| = 2$ Using Caucy's integral theorem [5+5+6]

5. (a) Show that when | z + 1 | < 1, $z^{-2} = 1 + \sum\limits_{n=1}^{\infty} (n+1)(z+1)^n$
   (b) Expand $f(z) = \frac{1}{z^2-z-6}$ about (i) z = -1 (ii) z = 1. [8+8]

6. (a) Find the poles and residues at each pole $\frac{2z+1}{(1-z^4)}$
   (b) Evaluate $\int \frac{\sin z}{z \cos z} dz$ where C is $|\,z\,| = \Pi$ by residue theorem. [8+8]

7. (a) Use method of contour integration to prove that $\int\limits_0^{2\pi} \frac{d\theta}{1+a^2-2aCos\theta} = \frac{2\pi}{1-a^2}$,0< a<1

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem. [8+8]

8. (a) Define conformal mapping. Let f(z) be an analytic function of z in a domain D of the z-plane and let f'(z)≠0 in D. Then show that w=f(z) is a conformal mapping at all points of D.

   (b) Find the bilinear transformation which maps the points (–i, o, i) into the point (–l, i, l) respectively. [8+8]

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