

II B.Tech I Semester Supplementary Examinations, November 2006
DISCRETE STRUCTURES AND GRAPH THEORY
 (Common to Computer Science & Engineering, Information Technology,
 Computer Science & Systems Engineering and Electronics & Computer
 Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions
 All Questions carry equal marks

1. (a) Show that RVS follows logically from premises. [8+8]
 $C \vee D, (C \vee D) \rightarrow H, H \rightarrow (A \wedge B) \rightarrow R \vee S$
 (b) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $R \vee P$ and Q .
2. (a) What are the properties of the relation $r = \{ (i, j) \mid |i - j| = 2 \}$ on the set $A = \{ 1, 2, 3, 4, 5, 6 \}$. [8+8]
 (b) Determine all the bijections from $\{ 1, 2, 3 \}$ on to $\{a, b, c, d\}$.
3. (a) Show that for any fixed k the relation given by $\{ \langle k, y \rangle \mid y > k \}$ is primitive recursive. [8+8]
 (b) Show that the sets of even numbers and odd numbers are both recursive.
4. (a) Are the graphs shown in the figure1 isomorphic ? [8+8]

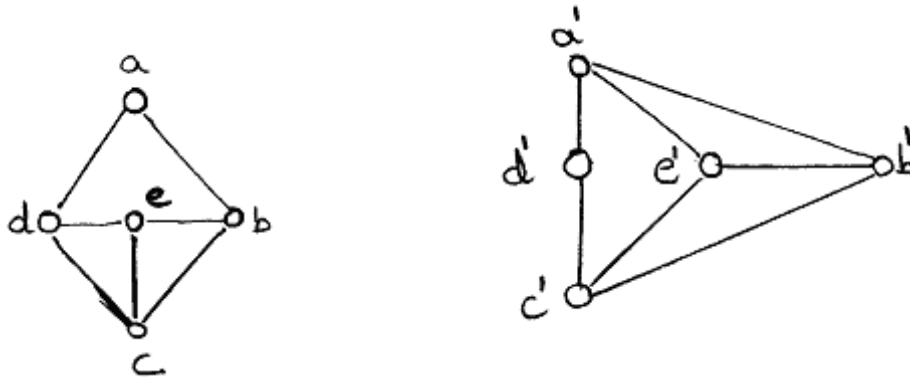


Figure 1:

- (b) Define isomorphism and give examples.
5. (a) Prove that the Kuratowski's second graph consisting of 6 vertices and 9 edges is non-planar.
 (b) State criteria to detect the planarity of a connected graph and give an example also. [8+8]

6. (a) Do an analysis of the Dijkstra-Prim minimum spanning tree algorithm, counting the number of times that an edge is considered for nodes added to the fringe, for updating edges to the fringenodes, or to pick the node to move from the fringe to the minimum spanning tree. [10]
- (b) What is the value of the postfix expression. [6]
 $723 * - 4 \uparrow 93 / + ?$
Give the solution steps.
7. (a) Explain the terms [10]
i. Disjunctive counting and
ii. Sequential counting.
- (b) How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8, and 9 if no repetitions are allowed? [6]
8. Solve the recurrence relation $a_n - 7a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$ for $n \geq 2$. [16]

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1. (a) Explain the Rules of inference. [10+6]
 (b) Demonstrate that “R” is a valid inference from the premises
 $P \rightarrow Q$, $Q \rightarrow R$, and P .
2. (a) Let $S = \{1, 2, 3, 4, 5\}$ and let $A = S \times S$. Define the following relation R on A
 such that $(a, b) R (a', b')$ if and only if $a \cdot b' = a' \cdot b$. [5+5+6]
 (b) Show that R is an equivalence relation.
 (c) Compute A/R .
3. (a) Let L be lattice. Then prove that $a \wedge b = a$ if and only if $a \vee b = b$. [8+8]
 (b) Define the dual of a statement in a lattice L . Why does the principle apply to L ?
4. (a) Define 1 - and 2- isomorphism with one example each. [6]
 (b) If G_1 and G_2 are two 1-isomorphic graphs then the rank of G_1 is equal to the
 rank of G_2 and the nullity of G_1 is equal to the nullity of G_2 . [10]
5. Show that a directed graph that contains an Euler circuit is strongly connected. [16]
6. (a) Describe the rules to convert a general tree to a binary tree. Illustrate with
 an example situation. [8+8]
 (b) Draw the binary tree for the following expression.
 $((x + 2) \uparrow 3) * (4 - (3 + x)) - 5$
 Find the infix, postfix, Prefix expressions.
7. (a) How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ where
 each $x_i \geq 2$? [8+8]
 (b) Find the number of distinct triples (x_1, x_2, x_3) of nonnegative integers satisfying
 the inequality $x_1 + x_2 + x_3 < 6$. [6]
8. Explain the Recurrence relation. What is its application in computer science ex-
 plain with suitable examples. [16]

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1. (a) Construct the truth tables for the following formula; [8+8]
 $(P \wedge Q) \vee (\neg P \wedge Q) \vee (P \wedge \neg Q) \wedge (\neg P \wedge \neg Q)$
 (b) Construct the truth tables of the following formula.
 $(P \rightarrow Q) \wedge (Q \rightarrow P)$

2. (a) Let the compatibility relation on a set $\{x_1, x_2, \dots, x_6\}$ be given by the following matrix. Draw the graph and find the maximal compatibility blocks of the relation. [8+8]

X_2	1				
X_3	1	1			
X_4	0	0	1		
X_5	0	0	1	1	
X_6	1	0	1	0	1
	X_1	X_2	X_3	X_4	X_5

 (b) Let R denote a relation on the set of ordered pairs of positive integers such that $\langle x, y \rangle R \langle u, v \rangle$ if and only if $xv = yu$. Show that R is an equivalence relation.

3. (a) Prove that if the function $f: A \rightarrow B$ has an inverse if and only if f is bijective.
 (b) Show that the set of positive N is a lattice with respect to the operations $a \vee b = \text{l.c.m.}(a, b)$ and $a \wedge b = \text{g.c.d.}(a, b)$, l.c.m. - least common multiple and g.c.d. - greatest common divisor. [8+8]

4. (a) Prove that the sequence 5,5,3,3,2,2 is graphic. Draw the graph [8+8]
 (b) Show that 5,5,3,3,2,2,2 form a graphical sequence

5. (a) Prove that the Kuratowski's second graph consisting of 6 vertices and 9 edges is non-planar.
 (b) State criteria to detect the planarity of a connected graph and give an example also. [8+8]

6. (a) Use depth first search to produce a spanning tree for the given graph. Choose node 'a' as the root for the figure1 [8+8]
 (b) Use kruskal's algorithm to find a minimum spanning tree for the weighted graph as shown in the figure2.

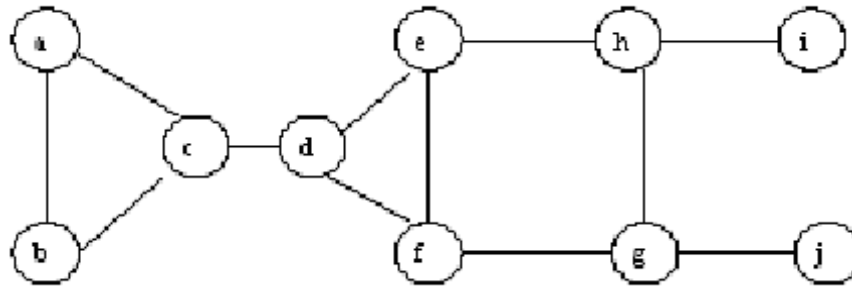


Figure 1:

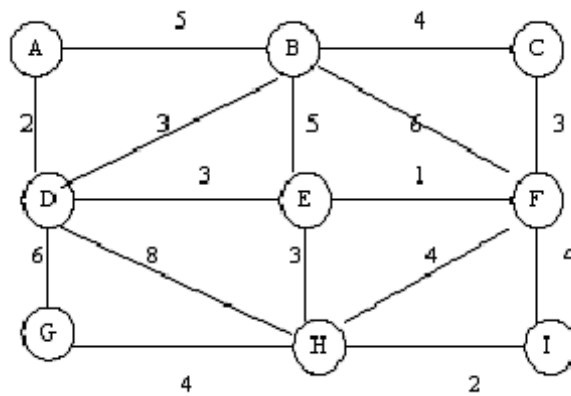


Figure 2:

7. (a) Explain the terms [10]
- i. Disjunctive counting and
 - ii. Sequential counting.
- (b) How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8, and 9 if no repetitions are allowed? [6]
8. Explain the methods of solving recurrence relations with suitable examples. [16]

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1. With reference to automatic theorem proving, show that SVR is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ [16]
2. (a) Given $S = \{1, 2, 3, \dots, 10\}$ and a relation R on S where $R = \{ \langle x, y \rangle / x + y + 10 \}$, what are the properties of the relation R ? [8+8]
 (b) Show that if $f \langle x, y \rangle$ defines the remainder upon the division of y by x , then it is a primitive recursive function.
3. (a) Let L a finite distributive lattice. Then prove that every element in L can be written uniquely (except for order) as the join of irredundant join-irreducible elements. [8+8]
 (b) Prove the independent laws for the elements of a lattice.
4. Let G be a complete directed graph. A non empty subset of the vertices of G is said to be an 'out classed group' if any edge joining a vertex in the subset and a vertex not in the subset is always directed from the latter to the former. Show that G has a directed circuit containing all the vertices, if there is no outclassed group of vertices. [16]
5. $K_{m,n}$ represents a complete bi partite graph. [5+5+6]
 (a) Is there a Hamiltonian circuit in $K_{4,6}$?
 (b) Is there a Hamiltonian path in $K_{4,5}$?
 (c) State a necessary and sufficient condition for the existence of Hamiltonian circuit in $K_{m,n}$.
6. (a) How many different directed trees are there with three nodes? How many different ordered trees are there with three nodes?
 (b) Show that in a complete binary tree, the total no. of edges is given by $2(n_t - 1)$, where n_t is the number of terminal nodes.
7. (a) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where for each; [10]
 i. $x_i \geq 0$;
 ii. $x_i \geq 1$;
 iii. $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$;

iv. $x_i \geq i$.

- (b) Six distinct symbols are transmitted through a communication channel. A total of 12 blanks are to be inserted between the symbols with at least 2 blanks between every pair of symbols. In how many ways can the symbols and blanks be arranged? [6]

8. Solve the recurrence relation $a_n - 7a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$ for $n \geq 2$. [16]
