

**II B.Tech I Semester Regular Examinations, November 2006**  
**MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE**  
 ( Common to Computer Science & Engineering, Information Technology  
 and Computer Science & Systems Engineering)

Time: 3 hours

Max Marks: 80

**Answer any FIVE Questions**  
**All Questions carry equal marks**

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1. (a) Let p,q and r be the propositions.  
 P: you have the flee  
 q: you miss the final examination.  
 r: you pass the course.  
 Write the following proposition into statement form.
  - i.  $P \rightarrow q$
  - ii.  $\neg p \rightarrow r$
  - iii.  $q \rightarrow \neg r$
  - iv.  $p \vee q \vee r$
  - v.  $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
  - vi.  $(p \wedge q) \vee (\neg q \wedge r)$
- (b) Define converse, contrapositive and inverse of an implication. [12+4]
2. Prove using rules of inference or disprove.
  - (a) Duke is a Labrador retriever  
 All Labrador retriever like to swim  
 Therefore Duke likes to swim.
  - (b) All even numbers that are also greater than 2 are not prime  
 2 is an even number  
 2 is prime  
 Therefore some even numbers are prime.  
 UNIVERSE = numbers.
  - (c) If it is hot today or raining today then it is no fun to snow ski today  
 It is no fun to snow ski today  
 Therefore it is hot today  
 UNIVERSE = DAYS. [5+6+5]
3. (a) Define a bijective function. Explain with reasons whether the following functions are bijective or not. Find also the inverse of each of the functions.
  - i.  $f(x) = 4x+2$ , A=set of real numbers
  - ii.  $f(x) = 3 + 1/x$ , A=set of non zero real numbers
  - iii.  $f(x) = (2x+3) \bmod 7$ , A= $\mathbb{N}_7$ .

- (b) Let  $f$  and  $g$  be functions from the positive real numbers to positive real numbers defined by  
 $f(x) = [2x]$   
 $g(x) = x^2$   
 Calculate  $f \circ g$  and  $g \circ f$ . [10+6]
4. (a) If  $G$  is the set of all positive rational numbers, then prove that  $G$  is an abelian group under the composition defined by  $\circ$  such that  $a \circ b = (ab)/3$  for  $a, b \in G$  with usual addition as the operation.
- (b) Let  $P(S)$  be the power set of a non empty set  $S$ . Let  $' \cap '$  be an operation in  $P(S)$ . Prove that associate law and commutative law are true for the operation  $\cap$  in  $P(S)$ . [12+4]
5. (a) How many ways are there to place 20 identical balls into 6 different boxes in which exactly 2 boxes are empty.
- (b) In how many ways can we partition 12 similar coins into 5 numbered nonempty batches. [16]
6. (a) Solve  $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$ .
- (b) Solve  $a_n + 3a_{n-1} - 10a_{n-2} = 0$ ,  $n \geq 2$ , given  $a_0 = 1$ ,  $a_1 = 4$  using generating functions. [8+8]
7. (a) Explain about the adjacency matrix representation of graphs. Illustrate with an example.
- (b) What are the advantages of adjacency matrix representation.
- (c) Explain the algorithm for breadth first search traversal of a graph. [5+3+8]
8. (a) Prove or disprove that the following two graphs are isomorphic. Figures 1, 2.

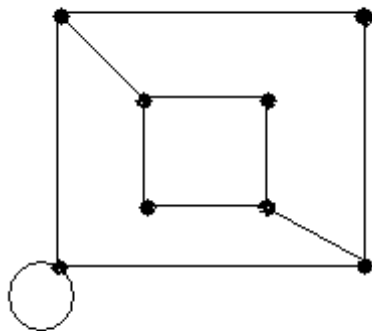


Figure 1:

- (b) Determine the number of edges in [8+8]
- i. Complete graph  $K_n$ ,

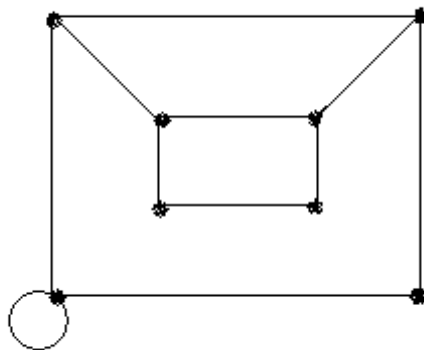


Figure 2:

- ii. Complete bipartite graph  $K_{m,n}$
- iii. Cycle graph  $C_n$  and
- iv. Path graph  $P_n$ .

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  - vi.  $(p \wedge q) \vee (\neg q \wedge r)$
- (b) Define converse, contrapositive and inverse of an implication. [12+4]
2. (a) Let P(x) denote the statement. "x is a professional athlete" and let Q(x) denote the statement "x plays soccer". The domain is the set of all people. Write each of the following proposition in English.
  - i.  $\forall x (P(x) \rightarrow Q(x))$
  - ii.  $\exists x (P(x) \wedge Q(x))$
  - iii.  $\forall x (P(x) \vee Q(x))$
- (b) Write the negation of each of these propositions, both in symbols and in words. [6+10]
3. (a) Determine whether the following relations are injective and/or surjective function. Find universe of the functions if they exist.
  - i.  $A = \{v, w, x, y, z\}$ ,  $B = \{1, 2, 3, 4, 5\}$   
 $R = \{(v, z), (w, 1), (x, 3), (y, 5)\}$
  - ii.  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{1, 2, 3, 4, 5\}$   
 $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)\}$
- (b) If a function is defined as  $f(x, n) \bmod n$ . Determine the
  - i. Domain of f
  - ii. Range of f
  - iii.  $G(g(g(g(7))))$  if  $g(n) = f(209, n)$ . [8+8]

4. S is any non-empty set. Verify whether the following are groups or not?  $(P(S), \cup)$  and  $(P(S), \cap)$  [16]
5. (a) Using the digits 1,3,4,5,6,8 and 9 how many 3-digit numbers can be formed  
 (b) How many 3-digit numbers can be formed if no digit is repeated?  
 (c) How many 3-digit numbers can be formed if 3 and 4 are adjacent to each other?  
 (d) How many 3-digit numbers can be formed if 3 and 4 are not adjacent to each other? [4+4+4+4]
6. (a) Assume that population of a village is 1000 at time  $n=0$  and that the increase from time  $n-1$  to  $n$  is 10 percent of the size at time  $n-1$ . Write a recurrence relation and an initial condition that define the population at time  $n$  and then solve the recurrence relation.  
 (b) Solve  $a_n - 7a_{n-1} + 10a_{n-2} = 0$ ,  $n \geq 2$ , given  $a_0 = 10$ ,  $a_1 = 41$  using generating functions. [6+10]
7. (a) Explain about the adjacency matrix representation of graphs. Illustrate with an example.  
 (b) What are the advantages of adjacency matrix representation.  
 (c) Explain the algorithm for breadth first search traversal of a graph. [5+3+8]
8. Find which of the following multi graphs have Euler paths, circuits or neither. [16]  
 Figure 1 , Figure 2
- (a)

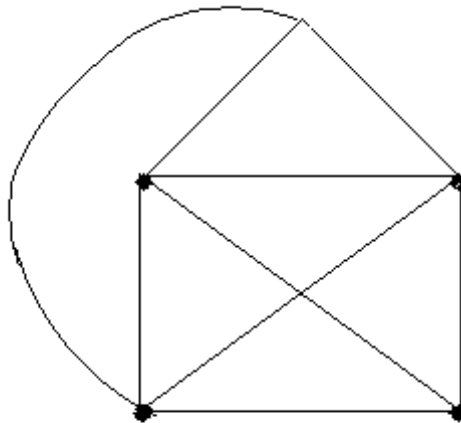


Figure 1:

(b)

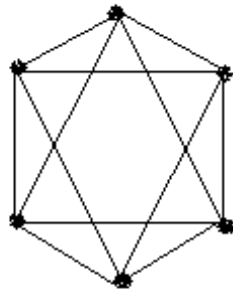


Figure 2:

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1. (a) Construct a truth table for each of the (easy) compound statements
  - i.  $(p \rightarrow q) \wedge (\neg p \rightarrow q)$
  - ii.  $p \rightarrow (\neg q \vee r)$
 (b) Write the negation of the following statements.
  - i. Jan will take a job in industry or go to graduate school.
  - ii. James will bicycle or run tomorrow.
  - iii. If the processor is fast then the printer is slow.
 (c) What is the minimal set of connectives required for a well formed formula.  

[8+6+2]
  
2. Prove using rules of inference or disprove.
  - (a) Duke is a Labrador retriever  
 All Labrador retrievers like to swim  
 Therefore Duke likes to swim.
  - (b) All even numbers that are also greater than 2 are not prime  
 2 is an even number  
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 It is no fun to snow ski today  
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 UNIVERSE = DAYS.  

[5+6+5]
  
3. (a) Determine whether the following relations are injective and/or surjective function. Find universe of the functions if they exist.
  - i.  $A = \{v, w, x, y, z\}$ ,  $B = \{1, 2, 3, 4, 5\}$   
 $R = \{(v, z), (w, 1), (x, 3), (y, 5)\}$
  - ii.  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{1, 2, 3, 4, 5\}$   
 $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 1)\}$
 (b) If a function is defined as  $f(x, n) \bmod n$ . Determine the
  - i. Domain of  $f$

- ii. Range of  $f$
- iii.  $G(g(g(g(7))))$  if  $g(n) = f(209, n)$ . [8+8]
4. (a)  $A$  is any non-empty set. Consider the group of bijections from  $A$  to  $A$  with binary composition  $(S, \cdot)$ . Show that  $(S, \cdot)$  is non abelian for  $n(A) > 2$ . [cardinality of  $A > 2$ ]
- (b) Prove that every homomorphic image of an abelian group is abelian. [8+8]
5. (a) How many ways are there to seat 10 boys and 10 girls around a circular table, if boys and girls seat alternatively
- (b) In how many ways can the digits 0,1,2,3,4,5,6,7,8 and 9 be arranged so that 0 and 1 are adjacent and in the order of 01. [16]
6. (a) Find recurrence relation for number of subsets of an  $n$ -element set.
- (b) Solve  $a_n - 5a_{n-1} + 6a_{n-2} = 2^n + n$ ,  $n \geq 2$ , given  $a_0 = 1$ ,  $a_1 = 1$  using generating functions. [4+12]
7. (a) Derive all six possible spanning trees from the graph, given below. Figure 1

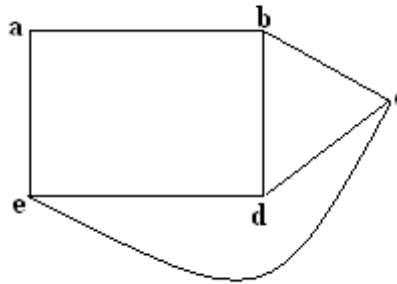


Figure 1:

- (b) State and prove Euler's formula for a connected planar graph. [8+8]
8. (a) Define the following graphs with one suitable examples for each graphs.
- Complement graph
  - Sub graph
  - Induced sub graph
  - Spanning sub graph.
- (b) Determine whether the following two graph are isomorphic or not. If isomorphic, give the one to one mapping. Figure 2, 3 [8+8]

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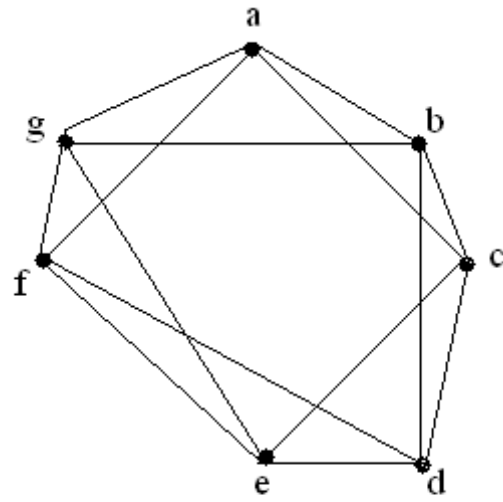


Figure 2:

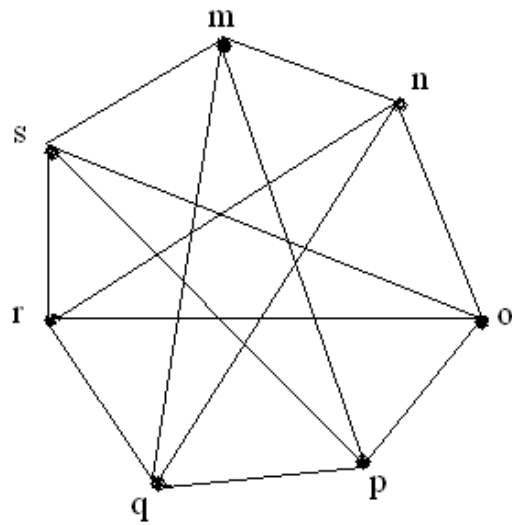


Figure 3:

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1. (a) Use De Morgan's laws to write the negation of each statement.
  - i. I want a car and a worth cycle.
  - ii. My cat stays outside or it makes a mess
  - iii. I've faller and I can't get up
  - iv. You study or you don't get a good grade.
- (b) Are  $(p \rightarrow q) \rightarrow r$  and  $p \rightarrow (q \rightarrow r)$  logically equivalent? Justify your answer by using the rules of logic to simplify both expressions and also by using truth tables. [8+8]
2. (a) Let  $P(x)$  denote the statement. "x is a professional athlete" and let  $Q(x)$  denote the statement "x plays soccer". The domain is the set of all people. Write each of the following proposition in English.
  - i.  $\forall x (P(x) \rightarrow Q(x))$
  - ii.  $\exists x (P(x) \wedge Q(x))$
  - iii.  $\forall x (P(x) \vee Q(x))$
- (b) Write the negation of each of these propositions, both in symbols and in words. [6+10]
3. (a) Consider  $f: Z^+ \rightarrow Z^+$  define by  $f(a) = a^2$ . Check if  $f$  is one-to-one and / or into using suitable explanation.
- (b) What is a partial order relation? Let  $S = \{x, y, z\}$  and consider the power set  $P(S)$  with relation  $R$  given by set inclusion. Is  $R$  a partial order.
- (c) Define a lattice. Explain its properties. [4+8+4]
4. (a) In group  $G$  for  $a, b \in G$ ,  $O(a) = 5$ ,  $b \neq e$  and  $ab a^{-1} = b^2$ . Show that  $O(b)$  is 31.
- (b) Fill in the blanks in the following composition table so that  $\circ$  is associative in  $S = \{a, b, c, d\}$ . [12+4]

$\circ$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d				

5. (a) Find the arrangements of letters of MISSISSIPPI

- (b) In howmany ways can 3 boys share 15 different sized apples if each takes 5. [16]
6. (a) Solve the recurrence relation  $a_n = a_{n-1} + \frac{n(n+1)}{2}$ ,  $n \geq 1$   
(b) Solve the recurrence relation  $a_r = a_{r-1} + a_{r-2}$  using generating function. [8+8]
7. (a) Explain about the adjacency matrix representation of graphs. Illustrate with an example.  
(b) What are the advantages of adjacency matrix representation.  
(c) Explain the algorithm for breadth first search traversal of a graph. [5+3+8]
8. (a) Prove that in any non directed graph there is an even numbers of vertices of add degree.  
(b) Distinguish between cycle and circuit. Give suitable examples for each.  
(c) What are the steps followed in discovering the isomorphism? [6+4+6]

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