

**II B.Tech I Semester Supplementary Examinations, November 2006**  
**MATHEMATICS-II**

( Common to Civil Engineering, Electrical & Electronic Engineering,  
 Mechanical Engineering, Electronics & Communication Engineering,  
 Computer Science & Engineering, Chemical Engineering, Electronics &  
 Instrumentation Engineering, Bio-Medical Engineering, Information  
 Technology, Electronics & Control Engineering, Mechatronics, Computer  
 Science & Systems Engineering, Electronics & Telematics, Metallurgy &  
 Material Technology, Electronics & Computer Engineering, Production  
 Engineering, Aeronautical Engineering, Instrumentation & Control  
 Engineering and Automobile Engineering)

Time: 3 hours

Max Marks: 80

Answer any FIVE Questions  
 All Questions carry equal marks

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1. (a) For what value of K the matrix

$$\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix} \text{ has rank 3.} \quad [8]$$

- (b) Find whether the following set of equations are consistent if so, solve them.

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + x_2 + x_3 - x_4 = 4$$

$$x_1 + x_2 - x_3 + x_4 = -4$$

$$x_1 - x_2 + x_3 + x_4 = 2.$$

[8]

2. Verify that the sum of eigen values is equal to the trace of A for the matrix

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \text{ and find the corresponding eigen vectors.} \quad [16]$$

3. (a) Show that the eigen value of an orthogonal matrix is of unit modulus. [5]

(b) Show that  $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$  is a Hermitian matrix. [6]

- (c) Prove that the matrix

$$\begin{bmatrix} \frac{-2}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} & \frac{2}{3} \end{bmatrix} \text{ is orthogonal.} \quad [5]$$

4. (a) Expand  $f(x) = e^{-x}$  as a Fourier series in  $(-1, 1)$ . [8]  
 (b) Expand  $x \sin x$  as sine series in  $0 < x < \pi$ . [8]
5. (a) Form the partial differential equation by eliminating the arbitrary function from  
 $z = x f(y) + y \phi(x)$ . [5]  
 (b) Solve the partial differential equation  $p^2 + q^2 = z^2(x^2 + y^2)$  [5]  
 (c) Solve the partial differential equation  $(3y + 2z) p + (4z - 3x) q + (zx + 4y) r = 0$ . [6]
6. A taut string of length  $L$  is fastened at both ends. The midpoint of the string is taken to a height “ $b$ ” and then released from rest in that position. Find the displacement of the string at any position  $x$  at any time  $t$ . [16]
7. (a) Find the finite cosine transform of  $\left\{ f(x) = \frac{\pi}{3} - x + \frac{x^2}{2\pi} \right\}$  [8]  
 (b) Find the Function if its Fourier sine transform is  $e^{-as}$ . [8]
8. (a) Find the Z-Transform of  $e^n \cosh a$  ( $a > 0$ ) [8]  
 (b) Find  $Z^{-1} \frac{2z^2 - 10z + 3}{(z-3)^2(z-2)}$  [8]

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1. (a) Find the rank of the matrix by reducing it to the normal form. [8]

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 3 & 2 & 2 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 5 & 6 \end{bmatrix}$$

- (b) Find whether the following set of equations are consistent if so, solve them.

$$x + y + z = 9, \quad 2x + 5y + 7z = 52, \quad 2x + y - z = 0.$$

[8]

2. Find the eigen values and the corresponding eigen vectors of the matrix  $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

[16]

3. (a) If  $X = x \cos \alpha - y \sin \alpha$ ,  $Y = x \sin \alpha + y \cos \alpha$  write the matrix A of the transformation and prove that  $A^{-1} = A^T$ , Hence write the inverse transformation.

[8]

- (b) A transformation from the variables  $x_1, x_2, x_3$  to  $y_1, y_2, y_3$  is given by  $Y = AX$  and another transformation from  $y_1, y_2, y_3$  to  $z_1, z_2, z_3$  is given  $Z = BY$  where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$$

Obtain the transformation from  $x_1, x_2, x_3$  to  $z_1, z_2, z_3$  [8]

4. (a) Obtain the Fourier series to represent  $f(x) = \frac{1}{4}(\pi - x)^2$ ,  $0 < x < 2\pi$ . [8]

- (b) Find the half range cosine series for the function  $f(x) = x^2$ , in  $0 \leq x \leq \pi$  and hence find the sum of the series  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  [8]
5. (a) Form the partial differential equation by eliminating the arbitrary function from  $z = f(x^2 - y^2)$ . [5]
- (b) Solve the partial differential equation  $(y + z)p + (z + x)q = x + y$  [6]
- (c) Solve the partial differential equation  $(x + pz)^2 + (y + qz)^2 = 1$  [5]
6. A bar 10cm long with insulated sides, has its ends A and B kept at  $20^\circ C$  and  $40^\circ C$ , respectively until steady state conditions prevail. The temperature at end A is then suddenly raised to  $50^\circ C$  and at the same instant, that of the end B reduced to  $10^\circ C$ . Find the subsequent temperature function  $u(x, t)$ . [16]
7. (a) Find the Fourier cosine transforms of  $e^{-ax} \cos ax$  [8]
- (b) Prove that the Fourier transform of the convolution of  $f(x)$  and  $g(x)$  is the product of their Fourier transforms. [8]
8. (a) If  $Z(u_n) = \left[ \frac{2z^2 + 3z + 4}{(z-3)^3} \right]$ . Find  $u_1$  and  $u_2$ . [8]
- (b) Find  $Z^{-1} \left[ \frac{8z^2}{(2z-1)(4z-1)} \right]$  [8]

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1. (a) Find the value of  $\lambda$  for which the system of equations [8]  
 $3x - y + 4z = 3$ ,  $x + 2y - 3z = -2$   
 $6x + 5y + \lambda z = -3$  will have infinite no of solutions and solve them with that  $\lambda$  value.

- (b) Find the rank of the matrix A by reducing it to the normal form

Where  $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$  [8]

2. For the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$  find  $A^4$  using cayley Hamilton theorem. [16]

3. (a) Define : [6]

- i. Spectral Matrix
- ii. Quadratic Form
- iii. Canonical form.

- (b) Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$  to the canonical form. [10]

4. (a) Find a Fourier series to represent  $x - x^2$  from  $x = -\pi$  to  $x = \pi$ . Hence show that  $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  [10]

- (b) Find the half range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases} \quad [6]$$

5. (a) Form the partial differential equation by eliminating the arbitrary constants a, b from  $z = ax + by + (a/b) - b$ . [5]

(b) Solve the partial differential equation  $x^2(y^2 - z^2)p + y^2(z^2 - x^2)q = z^2(x^2 - y^2)$ .  
[6]

(c) Solve the partial differential equation  $(2z - y)p + (x + z)q + (2x + y) = 0$ .  
[5]

6. A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form  $y = a \sin(\pi x/L)$  from which it is released at time  $t = 0$ . Find the displacement of any point at a distance x from one end at time t.  
[16]

7. Solve the diffusion equation  

$$\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2} \quad -\infty < x < \infty, t > 0 \quad \frac{\partial u}{\partial t}(0, t) = 0, t > 0.$$
 with the conditions,  
 $u(x, 0) = f(x)$  and  $\frac{\partial u}{\partial x}$ , u tends to zero as x tend to  $\pm\infty$  using Fourier Transforms.  
[16]

8. (a) Find  $Z[(n+1)^2]$  [6]  
 (b) Solve the difference equation using z-transforms  $u_{n+2} - 5u_{n+1} + 6u_n = 4^n$  given that  $u_0 = 0$   $u^1 = 1$ . [10]

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1. (a) Determine the rank of the matrix.

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \text{ by reducing it to normal form.} \quad [8]$$

- (b) Determine the values of
- $\lambda$
- for which the following set of equations may possess non-trivial solution and solve them in each case.

$$3x_1 + x_2 - \lambda x_3 = 0; \quad 4x_1 - 2x_2 - 3x_3 = 0; \quad 2\lambda x_1 + 4x_2 + \lambda x_3 = 0.$$

[8]

2. Diagonalize the matrix
- $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$
- and hence find
- $A^4$
- . [16]

3. (a) Identify the nature, index, signature of the quadratic form
- $2x_1x_2 + 2x_2x_3 + 2x_3x_1$
- . [10]

- (b) Prove that transpose of a unitary matrix is unitary. [6]

4. (a) Find a Fourier series to represent
- $x - x^2$
- from
- $x = -\pi$
- to
- $x = \pi$
- . Hence show that
- $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
- [10]

- (b) Find the half range sine series for the function

$$f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \frac{1}{2} < x < 1 \end{cases} \quad [6]$$

5. (a) Form the partial differential equation by eliminating the arbitrary function
- $z = f_1(y + 2x) + f_2(y - 3x)$
- . [5]

- (b) Solve the partial differential equation
- $p^2x^2 = z(z - qy)$
- [6]

- (c) Solve the partial differential equation
- $z = px + qy + \sqrt{(1 + p^2 + q^2)}$
- [5]

6. Two parallel edges and an end at right angles bound an infinitely long plane uniform plate. The width is  $\Pi$  this end is maintained at a temperature  $u_o$  at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state. [16]
7. Solve  $\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta t^2}$  for  $x > 0, t > 0$ , given that using Fourier transforms.
- (a)  $u(0, t) = 0$  for  $t > 0$
- (b)  $u(x, 0) = \begin{cases} 1 & \text{for } 0 < x < 1 \\ 0 & \text{for } x \geq 1 \end{cases}$  and
- (c)  $u(x, t)$  is bounded. [16]
8. (a) Find  $Z[C^n \cos(an)], n > 0$  [8]
- (b) Find  $Z^{-1} \frac{1}{(z-1/2)(z-1/3)}$  if  $1/3 < |z| < 1/2$  [8]

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