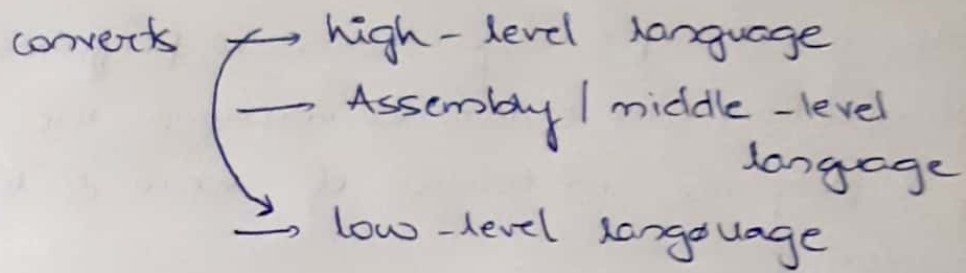


Compiler → used to find errors in programs



High → low level language is not a single step process

I Introduction  
Lexical Analysis

II Syntax Analysis

III SDT  
Intermediate Code Representation

IV Code generation

V Code Optimization

Course Objectives.

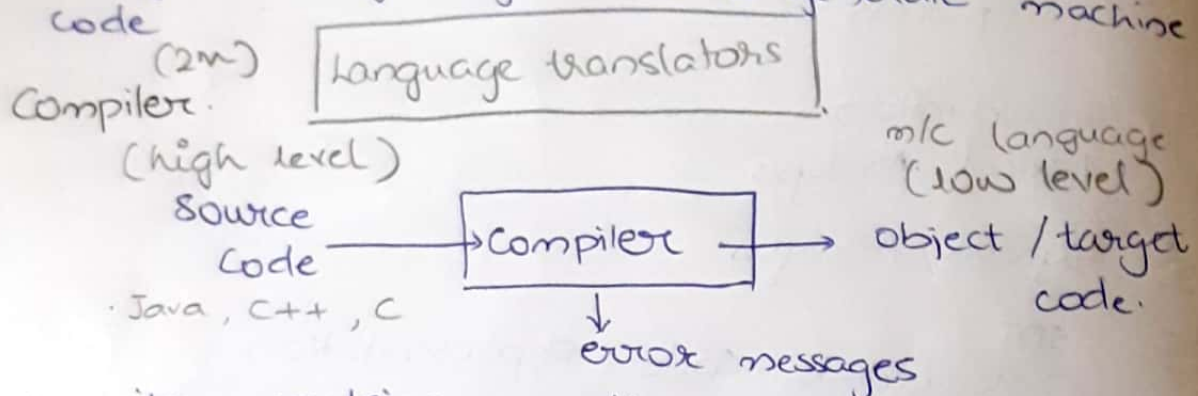
- To understand the various phases in the design of a compiler.
- To ~~design~~ understand the design of top-down and bottom-up parsers
- To understand syntax directed translation schemes
- To introduce lex and yacc tools
- To learn and develop algorithms, to generate code for a target machine

Course Outcomes:-

- Ability to design, develop and implement a compiler for any language
- Able to use lex and yacc tools for developing a scanner & a parser

- Able to design and implement LL and LR parsers
- Able to design algorithms to perform code optimization in order to improve the performance of a program, in terms of space & time complexity

- Ability to design algorithms to generate machine code



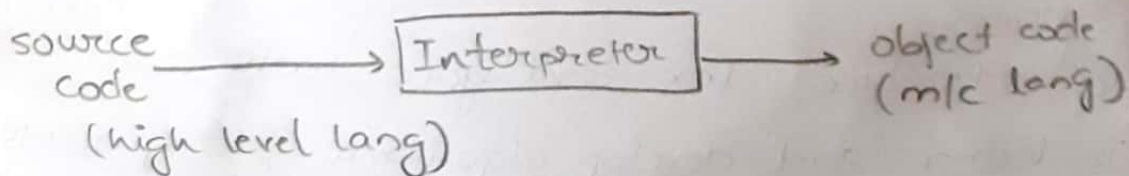
compiler contains a software programs that should satisfy the following properties.

Properties :-

- bug free
- proper machine code
- portable
- consists an optimized code
- execution fastly

- It converts the source code into an intermediate code (set of instructions)
- debugging is difficult
- execution is fast

Interpreter :



- An interpreter transforms / translating a source code to object code line by line
- execution is slow but debugging is easy
- languages ex:- Perl  
Python  
Ruby  
matlab



**Compiler :-** A compiler is a program that translates or converts a program written in high level language into equivalent program in low level language. And while doing so, it also produces error messages

(2M)

**Properties :-** (which are used in building a compiler)

- The program itself should be bug free
- It should translate proper machine code
- Execution must be fast
- Should be portable
- Should contain consistency in terms of optimized code
- It should provide error messages with proper line numbers

**Interpreter :-**

→ It is a program that translates or converts a program written in high level language to a <sup>equivalent</sup> program written in machine level language but line by line conversion takes place.

→ It produces error messages after translating every line and if an error occurs, unless it is cleared, the translation / conversion of the next line stops

**Differences btw compiler & interpreter**

Compiler	Interpreter
<ul style="list-style-type: none"> <li>• Compiler converts the entire source code at a time</li> <li>• Analysing the entire source code at a time</li> </ul>	<ul style="list-style-type: none"> <li>• Compiler Interpreter converts line by line</li> <li>• Analyzing line by line code is easy but</li> </ul>

is difficult but the execution is very fast

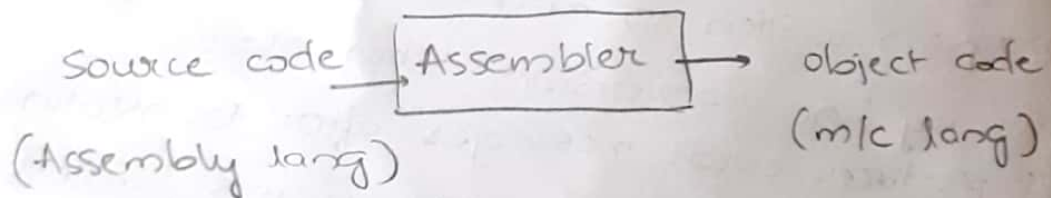
- Debugging is hard to implement
- Compiler can convert source code into intermediate form sometimes
- Ex: C, C++, Java

execution is slow

- Debugging is easy
- Interpreter doesn't do it.
- Ex:- Perl, Python, ruby, matlab.

Assembler:-

It is a software program which converts program written in assembly level language which contains symbolic instructions into equivalent machine language

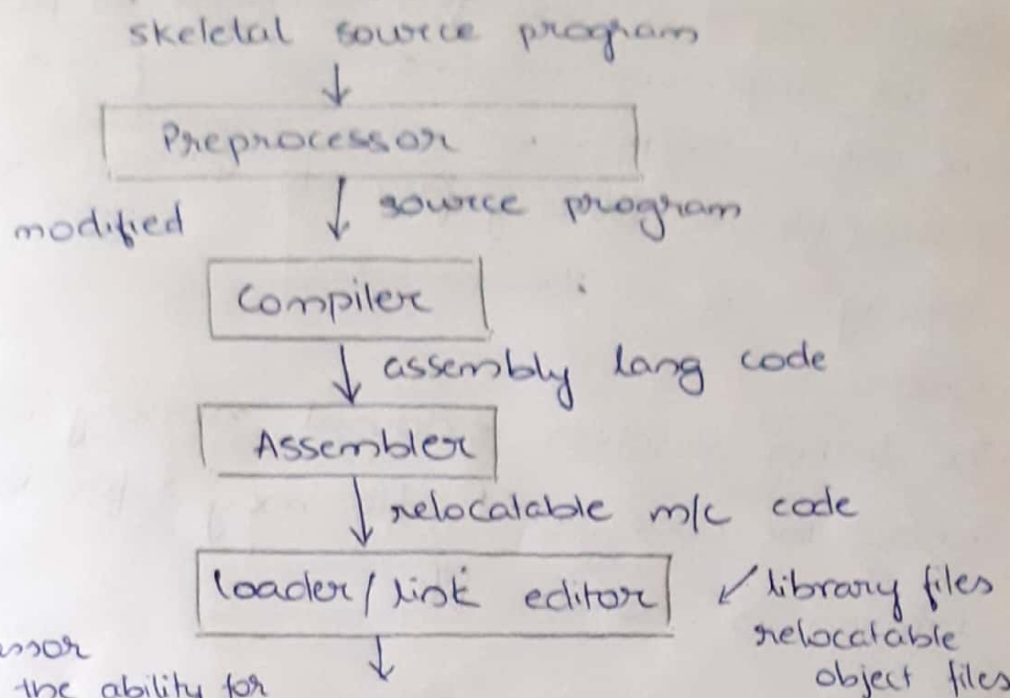


- language translators are software programs, that convert programs written in high level language / assembly language into equivalent machine language code

Language processing systems:-



## Language processing system :-



\* i.e.

preprocessor

provides the ability for inclusion of header files, macro expansions, conditional compilation & line control.

Preprocessor :-

- A source program may be divided into modules stored in separate files, the task of collecting the source program is sometimes included to a separate program called as a preprocessor
- \* It is a tool that produces input for compilers therefore it deals with micro processing augmentation, file inclusion, language extension etc.
- This modified source program is then fed to a compiler.

Compiler :-

- The compiler will produce an assembly lang program as its output because assembly language is easier to produce and easier to debug

Assembler :-

The assembly lang is then processed by a program called an assembler that produces relocatable machine code as its output



- Large programs are often compiled in pieces so the relocatable machine code may have to be linked together with other relocatable object files & library files into the code that actually runs on the machines

Linker :-

- The linker resolves external memory addresses where the code in 1 file may refer to a location in another file.

Loader :-

- The loader then puts together, all of the executable object files into memory for execution, it also calculates the total size of program & create memory place / space for it.
- Registers are used for fast access.

Note :-

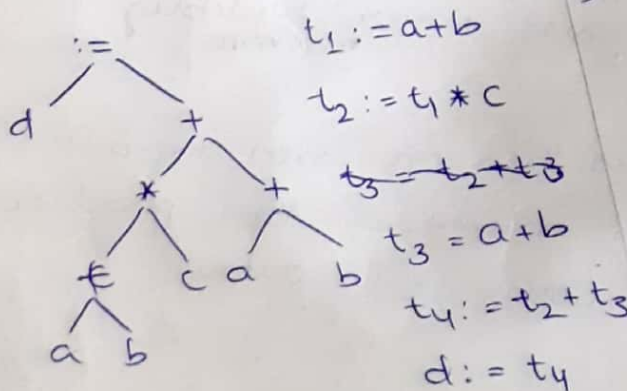
It initializes various registers to initiate execution

Structure of a compiler

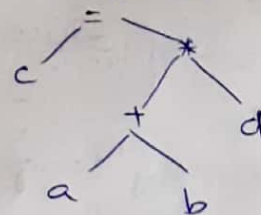
(or)

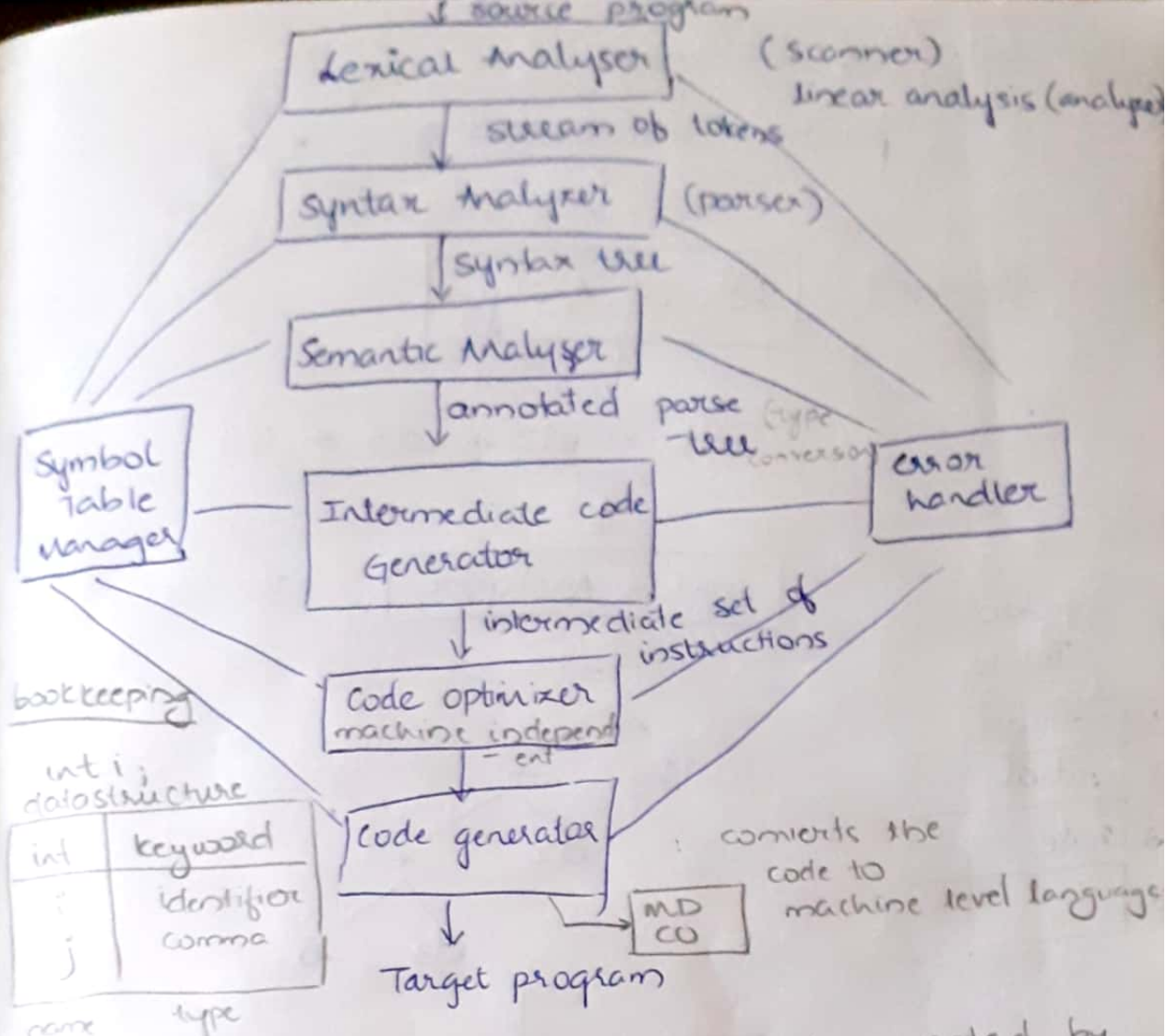
Phases of compiler / compilation

1)  $d := (a+b) * c + (a+b)$



2)  $c = (a+b) * d$



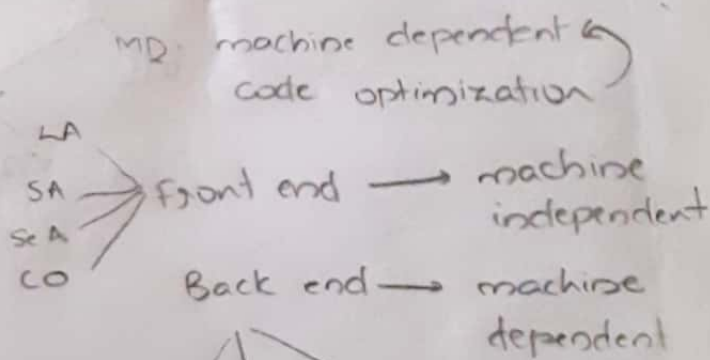


5) Code optimization :- Optimizes the code generated by the intermediate code generator.

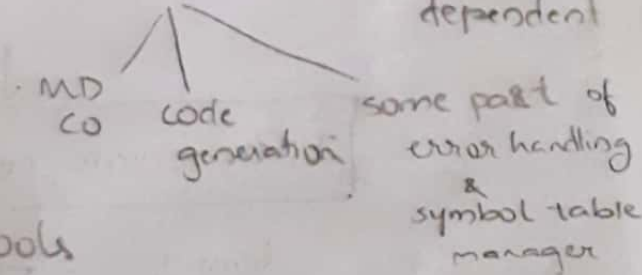
```

t1 = a + b;
t2 := t1 * c;
d := t1 + t2

```



1) LEX . It is a tool used by lexical analyzer to generate tools



# Translation of statement :-

position := initial + rate \* 60

Scanner

Lexical Analyser

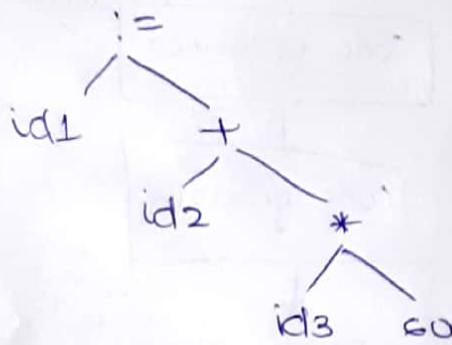
id1 := id2 + id3 \* 60

Syntax Analyser

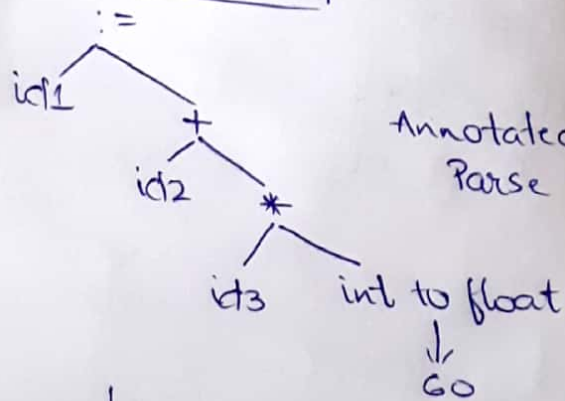
: Parser

Symbol Table

1	position
2	initial
3	rate
4	



Semantic Analyser



Annotated Parse tree

Intermediate code Generator

temp1 := ~~id3~~ \* (int to float) 60

temp2 := id3 \* temp1

temp3 := id2 + temp2

id1 := temp3



Code Optimizer

temp1 := id3 \* 60.0

id1 := id2 + temp1

Code Generator

LDF <sup>float</sup> R<sub>2</sub>, id3 → load id3 into R<sub>2</sub>  
MULF R<sub>2</sub>, R<sub>2</sub>, #60.0 → multiply R<sub>2</sub> & 60.0  
and store in R<sub>2</sub>  
LDF R<sub>1</sub>, id2 → load id2 into R<sub>1</sub>  
ADDF R<sub>1</sub>, R<sub>1</sub>, R<sub>2</sub> → add R<sub>1</sub>, R<sub>2</sub> and  
store in R<sub>1</sub>  
STF id1, R<sub>1</sub> → store R<sub>1</sub> into id1

Lexical analyzer:-

- It is the 1st phase of compilation
- It is also called as scanner because it work as text scanner
- This phase scans the source code as stream of characters and converts into meaningful lexemes.
- Lexical analyzer represents these lexemes into the form of tokens as

<tokenname, attribute name>

- Tokens can be keywords, identifiers, punctuation symbols, operators etc.

Syntax analyser

- The next phase in compiler is syntax analyser
- It is also called parser.

- It takes the tokens produced by lexical analyzer as input and generates parse stream (syntax stream)
- In this phase token arrangements are checked against the source code <sup>grammar</sup> i.e., the parser checks the expressions made by tokens are syntactically correct.

### Semantic analyzer :-

- It checks whether the parse tree constructed follows rules of language  
ex:- Assignment of value is b/w computable datatypes and adding string to an integer
- It checks if the given statement is syntactically correct or not.
- Also the semantic analyzer keeps the track of identifiers, their types & expressions and whether the identifiers are declared before use or not.
- It produces an annotated parse tree as output.

### Intermediate code generator :-

- After the semantic analysis, the compiler generates intermediate code of source code for target machine.
- It represents
- It is in b/w high level language & middle level language.



- This intermediate code should be generated in such a way that it makes it easier to translate into the machine code
- We have 3 different way for representing intermediate code

#### Code optimizer :-

- It is an optional phase of compilation & it does the optimization of the intermediate code
- This phase does the optimization by assuming as something that removes unnecessary code lines and arranges the sequence of statements in order to speed up the program execution ~~without~~ without wasting

#### Code generator :-

- In this phase, the code generator takes optimized representation of the intermediate code & maps it to target machine language.
- The code generator translates the intermediate code into a sequence of relocatable machine code.
- Sequence of instructions of machine code performs the task as the intermediate code would do.

#### Symbol table :-

- It is a datastructure maintained throughout all the phases of compiler.
- All the identifier names along with their types are stored here.
- The symbol table makes it easier for the compiler to quickly search the identifiers



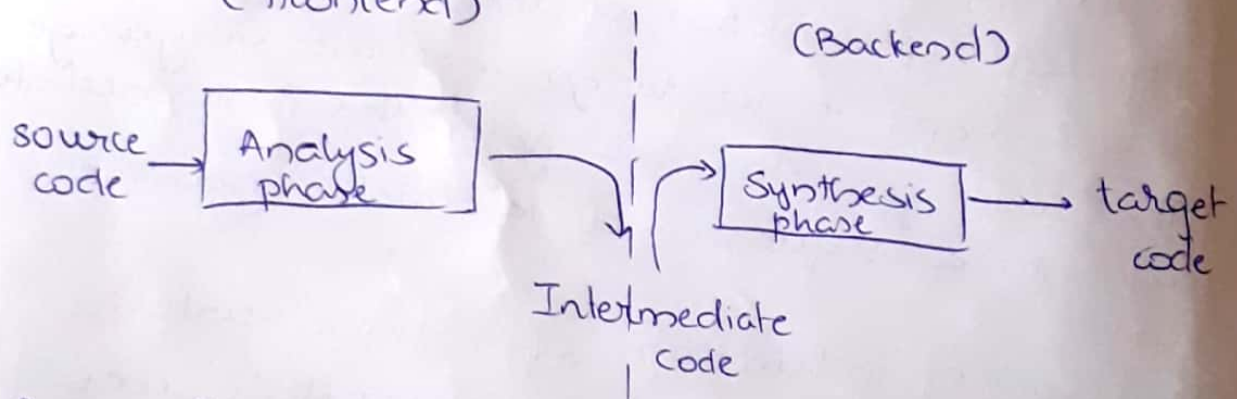
record & retrieve it.

- The symbol table is also used for scope management.

Error handler :-

- Error handler is invoked when an error in the source program is detected.
- It must warn the programmer by issuing a command and adjust the info being passed from phase to phase so that each phase can proceed.
- Both symbol table & error handler are routines interacts with all the phases of the compiler

Frontend & backend :-  
(Frontend)



A compiler can be broadly divided into 2 categories based on the way they can be compiler

- (i) Analysis phase :- It is known as the front end of the compiler. It reads the source program & divides it into core parts and then checks for lexical, grammatical & syntax errors.

- The analysis phase generates the intermediate representation of source programs which is fed as an input to synthesis phase
- It includes lexical analyser, syntax analyzer, semantic analyzer, intermediate code generator & some part of code optimization.
- It also includes creation of symbol-table, error handler that goes along with each of these phases.

### (ii) Synthesis phase :-

- It is known as back end of the compiler, it generates the target program with the help of intermediate code generation & symbol table

### Cross compiler :-

A compiler that runs on platform A is capable of generating an executable code for platform B is called cross compiler.

### Source to source compiler :-

A compiler that takes the source code of 1 programming language & translating it into source code of another programming language is called source to source compiler.

### Pass & phase :-

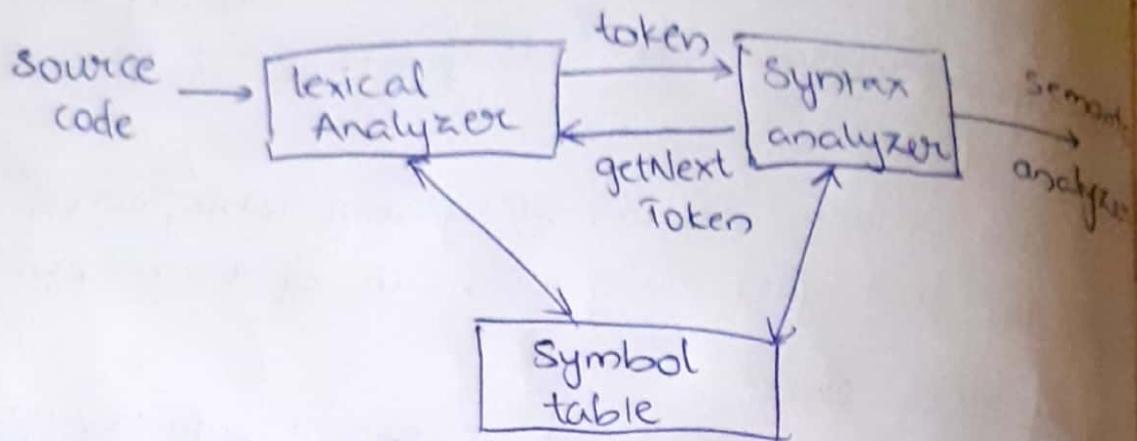
A pass refers to traversal of a compiler through an entire program.

Phase : A phase of compiler is a distinguishable stage, which takes input from previous stage, processes & yields output that can be used as



input for the next stage. A parse can have more than 1 phase.

## lexical Analysis:-



(A lexeme that matches a pattern is a token).  
token :- a smallest character which cannot be further divided.

lexeme :- a set of characters that forms a pattern / meaning.

pattern :- A rule describing a lexeme

- The secondary task of lexical analysis phase,
  - 1 task is stripping out of comments & white spaces in the form of blank tab & new line characters
  - 2nd task is correlating or associating error messages from the compiler with the source program. i.e, for eg:- the lexical analyzer may keep the track of new line characters seen, so that the line no. can be associated with any error message
  - lexical analyzer is the cascade of 2 methods
    - (i) Scanning :- for doing simple task
    - (ii) lexical analysis :-



→ A lexical analyzer deals with the terms, token, lexeme, pattern with specific meanings

token:- A sequence of characters having collective meaning.

lexeme:- A sequence of characters in a source pgm that is matched with a pattern for a token.

Pattern:- A pattern is a rule describing a set of lexemes that can represent a particular stmt in a source program

Token	Sample lexeme	Pattern
relop	<, <=, ==, <>, >, >=	< or <= or == or <> or > or >=
num	3.14, 0.142	any numeric const
literal	"core dumped"	any characters between " and "
id	pi, d3	letter followed by letter (or) digit any no. of times letter (letter/digit)*

Eg:- fortran:-

$$E = mc^2$$

$$E = m * c * * 2$$

<id, pointer to symbol table entry for E >

<assignop, >

<id, pointer to symbol table entry for m >

<mult-op, >

<id, pointer to symbol table entry for c >

<exp-op, >

<num, integer value 2 >

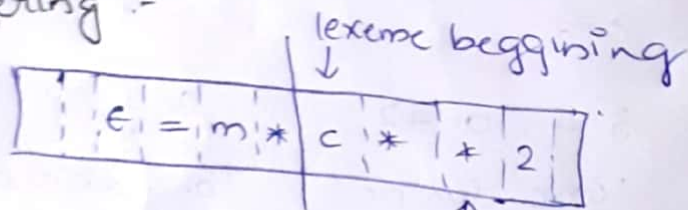
eg:  $f(a == f(x))$

→ This syntax error is not identified by the lexical analyzer but is done by the rest of the phases of compilation.

→ such type of syntax errors might be covered by following 4 techniques:

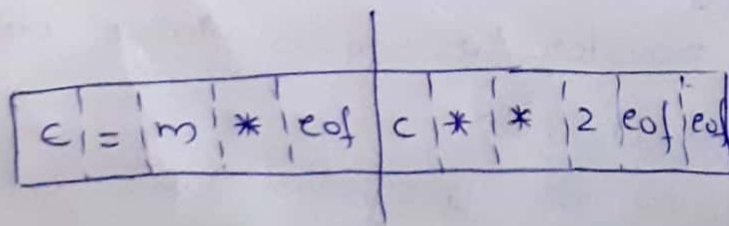
- (i) transposing of adjacent characters
- (ii) inserting a missing character
- (iii) Deleting an extra character.
- (iv) Replacing an incorrect character or string by the correct character/string.

Input buffering :-



- The lexical generator, in the order to scan the characters from ifp buffers maintains 2 pointers
  - (i) lexeme beginning
  - (ii) forward pointer
- Forward pointer is implemented until a lexeme is found
- End of the source program will be mentioned by EOF marker.

Sentinels :-





• code that represents the moving of forward pointer.

if forward at the end of the first half then begin

  reload second half;

  forward := forward + 1;

end

else if forward at the end of second half then begin

  reload first half;

  move forward pointer to beginning of first half;

end

else

  forward := forward + 1;

~~end~~

• code for the sentinals :

  forward := forward + 1;

  if forward ↑ := eof then

    if forward at the end of first half then begin

      reload second half;

      forward := forward + 1;

    end

    else if forward at the end of second half then begin

      reload first half;

      move forward pointer to beginning of first half.

    end

  else

  lexical analyzer stops

end

## Specification of tokens.

- Tokens can be described with the help of regular expression

Alphabet: The term alphabet or character class denotes any finite set of symbols. Symbols can be letters & characters.

String:- A string over some alphabet is a finite sequence of symbols drawn from that alphabet.

- It is also called as sentence/word.
- length of the string is represented by  $|s|$ .
- empty string is denoted by  $\epsilon$  is a special string of length 0 (zero).

Prefix & suffix of the string :-

Eg:- CSE3      prefix: CS  
                                suffix: E3  
                                substring: SE

- A string not containing  $\epsilon$  and it isn't a same string, then it is called a proper prefix/suffix.

eg:- CS, E3

- If substring is not continuous, then it is subset.

eg:- CSE

$\epsilon\epsilon \rightarrow$  subset of CSE3

Language :-

It is a set of strings performed on alphabets if L & M are the languages, following operations can be performed.



Regular expression :-  $\Sigma$  is symbol of alphabet (set of symbols)  
 Regular expression is defined over an alphabet. ' $\Sigma$ ' in the following way where  $\Sigma$  & the sets are defined as follows.

- (1)  $\phi$  is a R.E. and denotes an empty set.
- (2)  $\epsilon$  is a regular expression and denotes  $\{\epsilon\}$  i.e. a set containing an empty string.
- (3) If 'a' is a symbol in  $\Sigma$  then 'a' is a regular expression that denotes  $\{a\}$  i.e. the set containing string 'a'.
- (4) If  $r$  and  $s$  are regular expressions denoting the languages  $L(r)$  and  $L(s)$  then  $(r)|(s)$  is a R.E. denoting  $L(r) \cup L(s)$ . Similarly  $r.s$  is a R.E., denoting  $L(r).L(s)$ . Similarly  $(r)^*$  is a R.E. denoting  $L(r)^*$  is a r.e.

Axioms / properties of R.E. :

Axioms	description
1) $r s = s r$	is commutative
2) $r(s t) = (r s)t$	is associative
3) $rs = sr$	concatenation is commutative.
4) $r(st) = (rs)t$	concatenation is associative
5) $r(s t) = rs rt$	concatenation
$(s t)r = (sr)(tr)$	distributive over $r$
6) $\left. \begin{matrix} \epsilon r = r \\ r \epsilon = r \end{matrix} \right\}$	where $\epsilon$ is an identity element for concatenation
7) $r^{**} = r^*$	* is idempotent

→ unary \* is having highest precedence

→ concatenation - 2nd } associativity  
L to R



LEX tool :- It is an automated tool used by the lexical analyzer to generate the tokens

lex.l → lex compiler → lex.yy.c

lex.yy.c → C compiler → a.out

Input stream → a.out → sequence of tokens  
lex program syntax:  
→ declarations

% %

→ translation rules

% %

auxiliary procedures

P<sub>1</sub> { action 1 }

P<sub>2</sub> { action 2 }

⋮

P<sub>n</sub> { action n }

write a lex program  
To identify the ~~keywords~~ identifiers.

```
% { #include <stdio.h>
```

```
% }
```

```
digit [0-9]
```

```
letter [A-Z a-z]
```

```
% %
```

```
{ letter } ( { letter / digit } ) *
```

```
{ printf (" %s is an identifier ", yytext); }
```

```
% %
```

```
main()
```

```
{  
    yylex();
```

```
}
```

o/p :- cse is an identifier.

Q write a lex program to recognize a keyword

```
% { #include <stdio.h>
```

```
% }
```

```
keyword int / float / char / for / while / do
```

```
% {
```

```
{ keyword } { printf (" %s is a keyword ", yytext) ; }
```

```
% }
```

```
main ()
```

```
{ yylex ();
```

```
}
```

Q write a lex program to recognize arithmetic operations, relational operators, logical & bitwise operators

```
% { #include <stdio.h>
```

```
% }
```

```
operators * / + / - / / / < / <= / <> / > / >= / == /  
" / && / ! / | / & / ^
```

```
% {
```

```
{ operators } { printf (" %s is an operator ", yytext) ; }
```

```
% }
```

```
main ()
```

```
{ yylex ();
```

```
}
```



```
1. { # include <stdio.h>
```

```
1. }
```

```
arithmetic *|+|-|/
```

```
relational <|<=|<>|>|>|=|==
```

```
logical |||&&|!
```

```
bitwise ||&|^
```

```
1. 1.
```

```
{ arithmetic } { printf( ".s is an arithmetic operator",  
yytext ); }
```

```
{ relational } { printf( ".s is an relational operator",  
yytext ); }
```

```
{ logical } { printf( ".s is a logical operator",  
yytext ); }
```

```
{ bitwise } { printf( ".s is a bitwise operator",  
yytext ); }
```

```
1. 1.
```

```
main ( )
```

```
{
```

```
yytex ( );
```

```
}
```

metacharacters :-

\* 0-9 (0 or more)

+ 1-9 (1 or more)

? 0/1

[ ] - character class

( ) - enclosing of RE is one format

if . keyword is

- ^ to specify the beginning of a character string.
- \$ to specify the end of a character string
- [^] to specify all string, not starting with (^) a character.
- " " iterator, a string
- \ esc character
- | or symbol (pipe)

yyerror() :- It is used to display the error msgs.

yyin() :- It is variable used to store i/p source program

yyout() :- It is used to store the o/p file.

yylen() :- It is used to store length of the string

yywrap() :- The function returns 2 values  
If it returns zero(0) . It continues the scanning

If it returns 1 It shows it reaches the end.

~~yyval~~  
yyval() :- This variable is used to store the next generated token.

### Finite Automata:

$$M = (Q, \Sigma, \delta, q_0, F)$$

A finite automata can be represented by a 5 tuple notation

$$M = (Q, \Sigma, \delta, q_0, F)$$

where,

- Q = finite non-empty set of states
- $\Sigma$  = finite set of non-empty set of i/p called as input alphabet
- $\delta$  = is a transition fn which maps

$$Q \times E \rightarrow Q$$

$q_0$  is the initial state

$F$  is the final state where  $F \subseteq Q$

General definition of FA:

An automata is defined as a system where energy materials and information are transformed and used for performing some function without direct participation of humans.

Non-deterministic finite automata

A DFA which has ambiguity is called NFA.

A FA which allows 0, 1 or more transitions from a state on the same i/p symbol is called

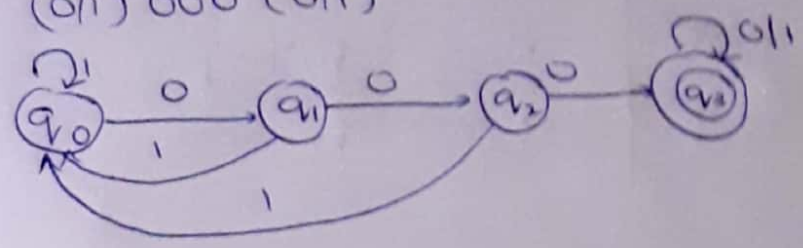
NFA



Q. Draw a DFA to accept the strings of 0's & 1's having 3 consecutive 0's.

$(0/1)^* 000 (0/1)^*$

DFA:

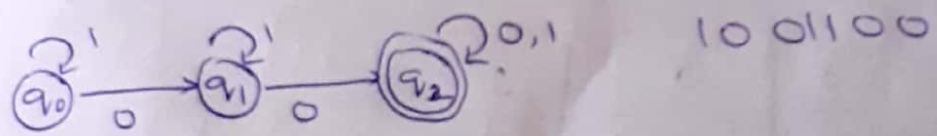


$\delta$	0	1
$q_0$	$q_1$	$q_0$
$q_1$	$q_2$	$q_0$
$q_2$	$q_3$	$q_0$
$q_3$	$q_3$	$q_0$

Transition table

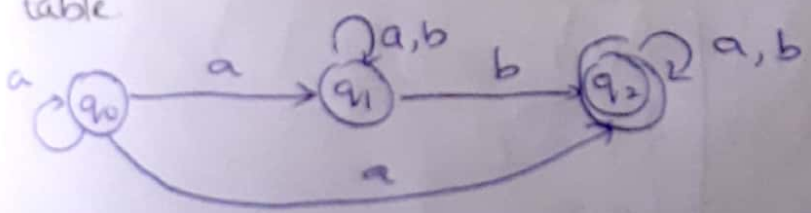
Q. Draw a DFA to accept the strings of 0's & 1's having atleast 2 zeroes.

$00, (0/1)^* 0 (0/1)^* 0 (0/1)^*$



$\delta$	0	1
$q_0$	$q_1$	$q_0$
$q_1$	$q_2$	$q_1$
$q_2$	$q_2$	$q_0$

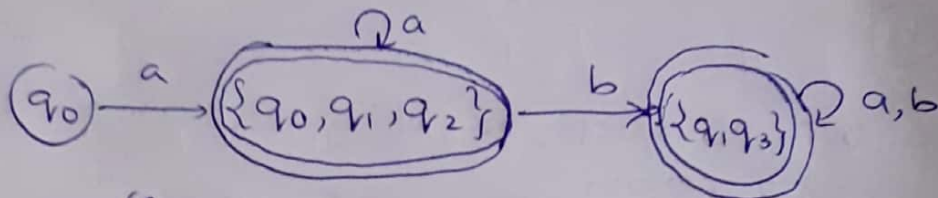
Q. Convert NFA to DFA and then draw transition table.



$\delta$	a	b
$q_0$	$\{q_0, q_1, q_2\}$	$\phi$
$q_1$	$q_1$	$\{q_1, q_2\}$
$q_2$	$q_2$	$q_2$

$\delta$	a	b
$q_0$	$\{q_0, q_1, q_2\}$	$\phi$
$q_1$	$q_1$	$\{q_1, q_2\}$
$q_2$	$q_2$	$q_2$

$\delta$	a	b
$q_0$	$\{q_0, q_1, q_2\}$	$\phi$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$



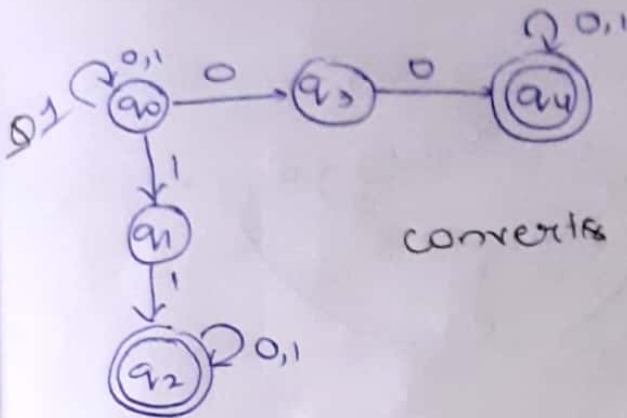
Q2)  $M = (\{p, q, r, s\}, \{0, 1\}, \delta, p, \{s\})$

$\delta$	0	1
p	$\{p, q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	$\phi$
s	$\{s\}$	$\{s\}$

$\delta$	0	1
p	$\{p, q\}$	$\{p\}$
$\{p, q\}$	$\{p, q, r\}$	$\{p, r\}$
$\{p, q, r\}$	$\{p, q, r, s\}$	$\{p, r\}$
$\{p, r\}$	$\{p, q, s\}$	$\{p\}$
$\{p, q, r, s\}$	$\{p, q, r, s\}$	$\{p, r, s\}$
$\{p, q, s\}$	$\{p, q, s\}$	$\{p, r, s\}$

$\{p, q, r, s\}$	$\{p, q, s\}$	$\{p, s\}$
$\{p, s\}$	$\{p, q, s\}$	$\{p, s\}$

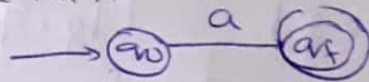
Draw DFS



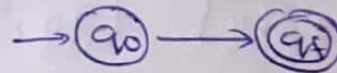
convert it to DFS.

R.E. to  $\epsilon$ -NFA

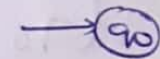
1)  $r = a$



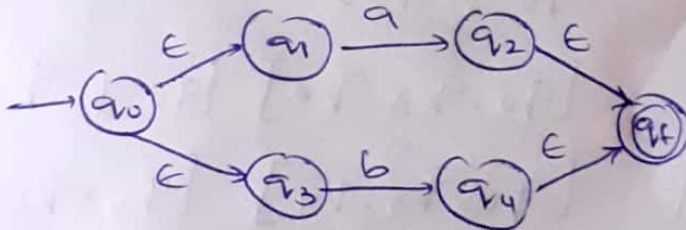
2)  $r = \phi$



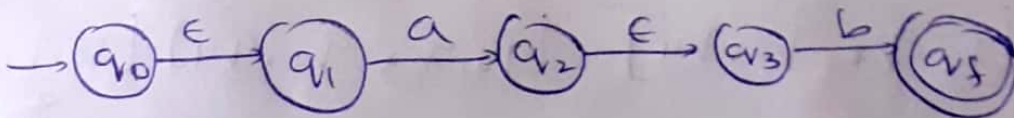
3)  $r = \epsilon$



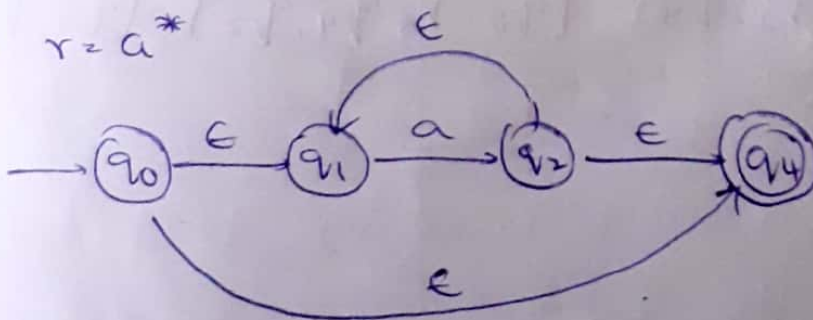
4)  $r = a/b$  (or)  $(a+b)$



5)  $r = ab$

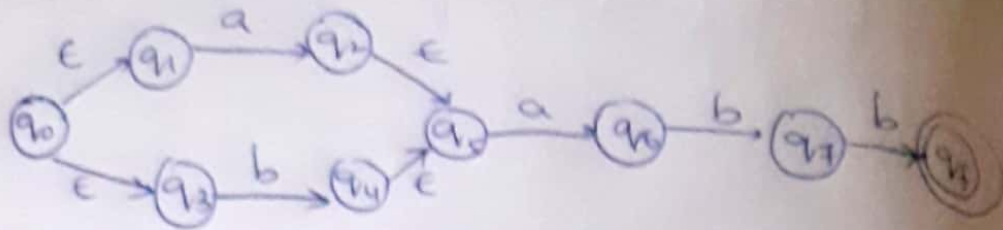


6)  $r = a^*$

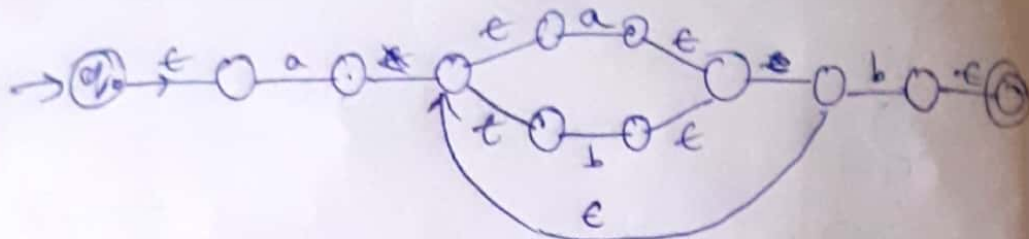




Obtain CNFA for the regular expression  
 $(ab)^* abb$



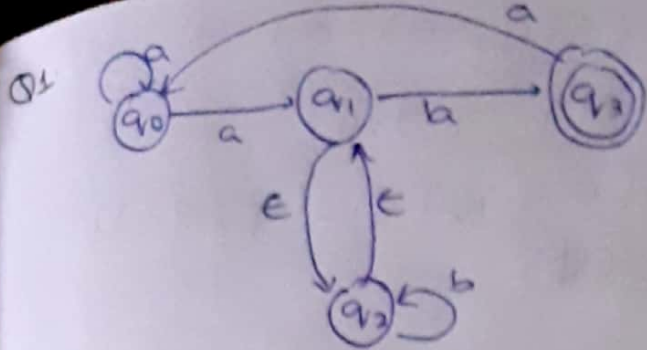
Obtain CNFA for the R.E  $a(a+b)^* b$



1)

$\delta$	$\emptyset$	$\perp$
$q_0$	$\{q_0, q_3\}$	$\{q_0, q_1\}$
$\{q_0, q_3\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_3\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_3, q_4\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_1, q_4\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_3, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_4\}$	$\{q_0, q_3, q_4\}$	$\{q_0, q_1, q_2, q_4\}$
$\{q_0, q_3, q_2\}$	$\{q_0, q_3, q_4, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_3, q_2, q_4\}$	$\{q_0, q_3, q_2, q_4\}$	$\{q_0, q_1, q_2, q_4\}$
$\{q_0, q_3, q_2, q_4\}$	$\{q_0, q_3, q_4, q_2\}$	$\{q_0, q_1, q_2, q_4\}$

$\epsilon$ -NFA to NFA :-



$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_3) = \{q_3\}$$

$$\begin{aligned} \delta(q_0, a) &= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), a)) \\ &= \epsilon\text{-closure}(\delta(q_0, a)) \\ &= \epsilon\text{-closure}(q_1) \\ &= \{q_0, q_1, q_2\} \end{aligned}$$

$$\begin{aligned} \delta(q_0, b) &= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), b)) \\ &= \epsilon\text{-closure}(\delta(q_0, b)) \\ &= \epsilon\text{-closure}(\phi) \\ &= \phi \end{aligned}$$

$$\begin{aligned} \delta(q_1, a) &= \epsilon\text{-closure}(\delta(q_1, \epsilon), a) \\ &= \epsilon\text{-closure}(\delta(q_1, q_2), a) \\ &= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a)) \\ &= \epsilon\text{-closure}(\phi \cup \phi) \\ &= \phi \end{aligned}$$

$$\begin{aligned} \delta(q_1, b) &= \epsilon\text{-closure}(\delta(q_1, \epsilon), b) \\ &= \epsilon\text{-closure}(\delta(q_1, q_2), b) \\ &= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b)) \\ &= \epsilon\text{-closure}(q_3, q_2) \\ &= \{q_1, q_2, q_3\} \end{aligned}$$

$$\delta(q_2, a) = \epsilon\text{-closure}(\delta(q_2, \epsilon), a)$$

$$= \epsilon\text{-closure}(\delta(q_1, q_2), a)$$

$$= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, \epsilon))$$

$$= \epsilon\text{-closure}(\phi)$$

$$= \phi$$

$$\delta(q_2, b) = \epsilon\text{-closure}(\delta(q_2, \epsilon), b)$$

$$= \epsilon\text{-closure}(\delta(q_1, q_2), b)$$

$$= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$$

$$= \epsilon\text{-closure}(q_3, q_2)$$

$$= \{q_1, q_2, q_3\}$$

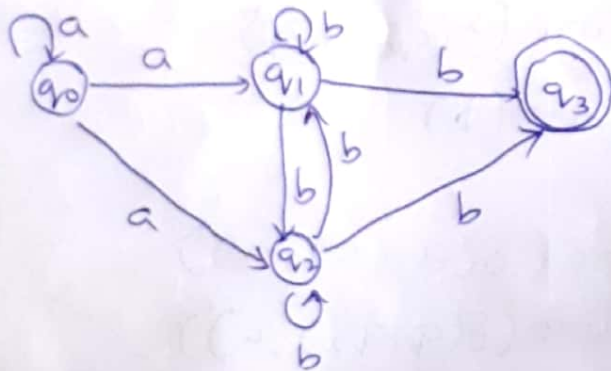
$$\delta(q_3, a) = \epsilon\text{-closure}(\delta(q_3, \epsilon), a)$$

$$= \epsilon\text{-closure}(\delta(q_3, a))$$

$$= \epsilon\text{-closure}\{q_0\}$$

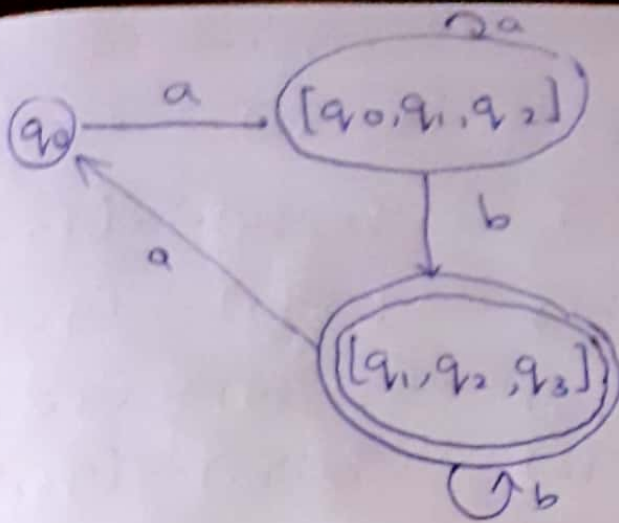
$$= \{q_0\}$$

$$\delta(q_3, b) = \phi$$

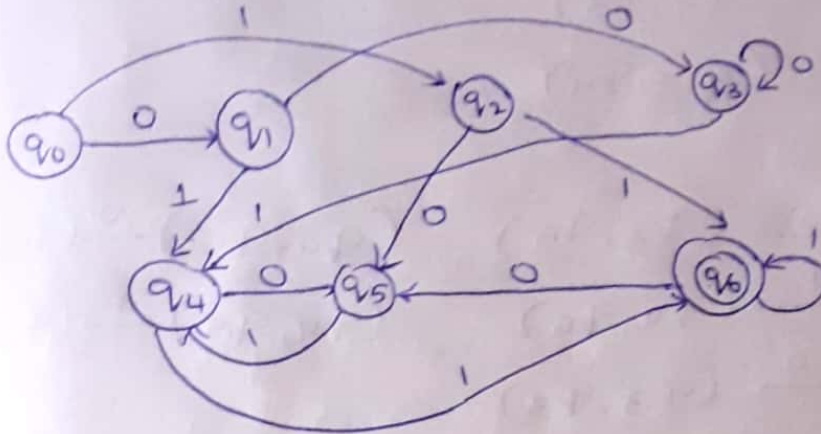


$\delta$	a	b
$q_0$	$\{q_0, q_1, q_2\}$	$\phi$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2, q_3\}$
$\{q_1, q_2, q_3\}$	$\{q_0\}$	$\{q_1, q_2, q_3\}$





Q2.



Transition table :-

$\delta$	0	1
$q_0$	$q_1$	$q_2$
$q_1$	$q_3$	$q_4$
$q_2$	$q_3$	$q_5$
$q_3$	$q_3$	$q_4$
$q_4$	$q_5$	$q_6$
$q_5$	$q_5$	$q_6$
$q_6$	$q_5$	$q_6$

$$(q_0, q_1) \xrightarrow{0} (q_1, q_3)$$

$$(q_0, q_1) \xrightarrow{1} (q_2, q_4)$$

$$(q_0, q_2) \xrightarrow{0} (q_1, q_5)$$

$$(q_0, q_2) \xrightarrow{1} (q_2, q_6)$$

$$(q_0, q_3) \xrightarrow{0} (q_1, q_3)$$

$$(q_0, q_3) \xrightarrow{1} (q_2, q_4)$$

$$(q_0, q_4) \xrightarrow{0} (q_1, q_5)$$

$$(q_0, q_4) \xrightarrow{1} (q_2, q_6)$$

$$(q_0, q_5) \xrightarrow{0} (q_1, q_3)$$

$$(q_0, q_5) \xrightarrow{1} (q_2, q_4)$$

$$(q_1, q_2) \xrightarrow{0} (q_3, q_5)$$

$$(q_1, q_2) \xrightarrow{1} (q_4, q_6)$$

$$(q_1, q_3) \xrightarrow{0} (q_3, q_3)$$

$$(q_1, q_3) \xrightarrow{1} (q_4, q_4)$$

$$(q_1, q_4) \xrightarrow{0} (q_3, q_5)$$

$$(q_1, q_4) \xrightarrow{1} (q_3, q_6)$$

$$(q_1, q_5) \xrightarrow{0} (q_3, q_3)$$

$$(q_1, q_5) \xrightarrow{1} (q_4, q_4)$$

$$(q_2, q_3) \xrightarrow{0} (q_5, q_3)$$

$$(q_2, q_3) \xrightarrow{1} (q_6, q_6)$$

$$(q_2, q_4) \xrightarrow{0} (q_5, q_5)$$

$$(q_2, q_4) \xrightarrow{1} (q_6, q_6)$$

$$(q_2, q_5) \xrightarrow{0} (q_3, q_3)$$

$$(q_2, q_5) \xrightarrow{1} (q_6, q_4)$$

$$(q_3, q_4) \xrightarrow{0} (q_3, q_5)$$

$$(q_3, q_4) \xrightarrow{1} (q_4, q_6)$$

$$(q_3, q_5) \xrightarrow{0} (q_3, q_3)$$

$$(q_3, q_5) \xrightarrow{1} (q_4, q_4)$$

$$(q_4, q_5) \xrightarrow{0} (q_5, q_2)$$

$$(q_4, q_5) \xrightarrow{1} (q_6, q_4)$$

$$(q_0, q_1) \quad (q_0, q_3)$$

$$(q_0, q_5) \quad (q_1, q_3)$$

$$(q_2, q_4)$$

$$(q_3, q_5) \quad (q_1, q_5)$$

$$(q_2, q_5) \quad (q_6)$$

q <sub>1</sub>	✓					
q <sub>2</sub>	x	x				
q <sub>3</sub>	✓	✓	x			
q <sub>4</sub>	x	x	✓	x		
q <sub>5</sub>	✓	✓	x	✓	x	
q <sub>6</sub>	x	x	x	x	x	x
	q <sub>0</sub>	q <sub>1</sub>	q <sub>2</sub>	q <sub>3</sub>	q <sub>4</sub>	q <sub>5</sub>

Rules :-

1. If one of the resultant states is FS then final state

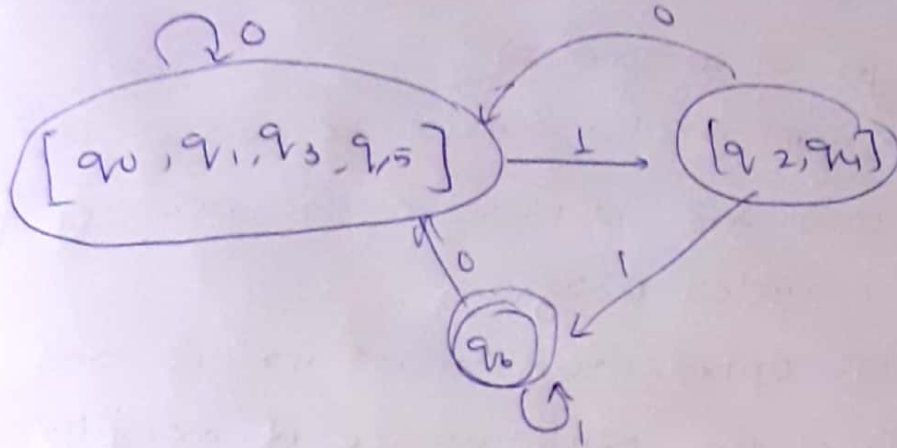
otherwise ✓

2. If both states are the same equivalent final state then ✓

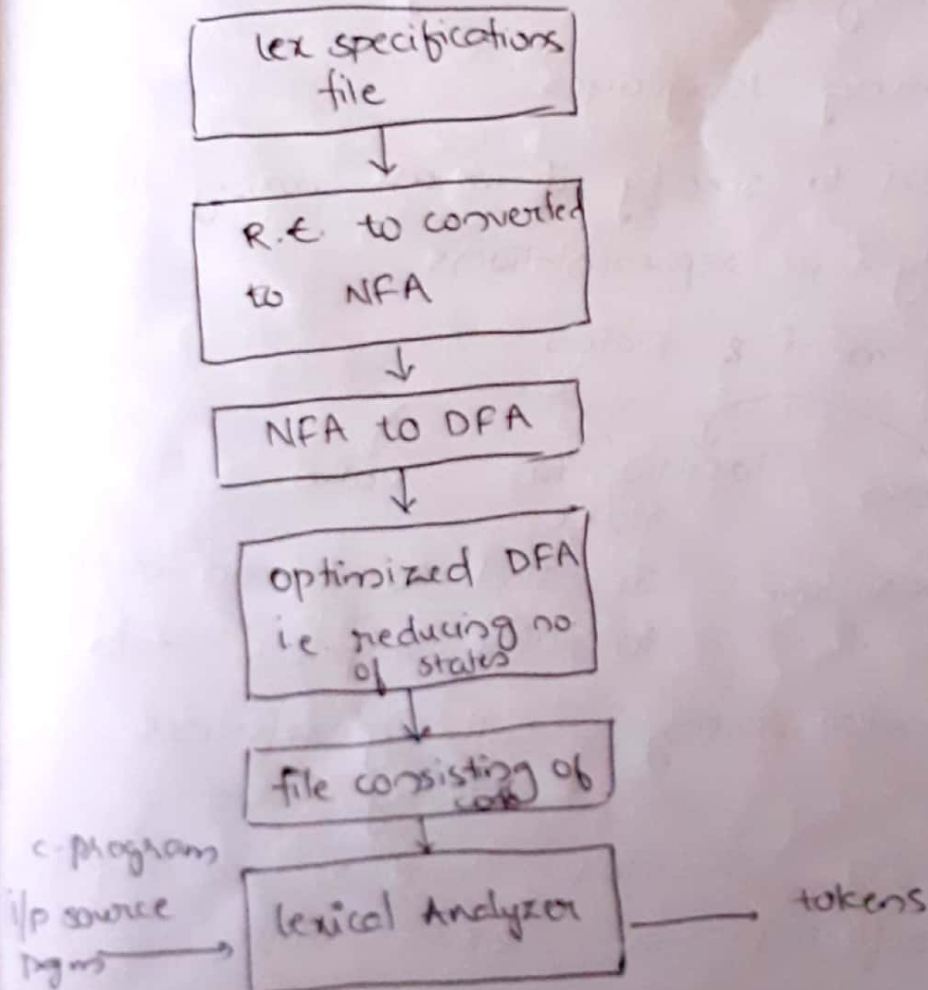
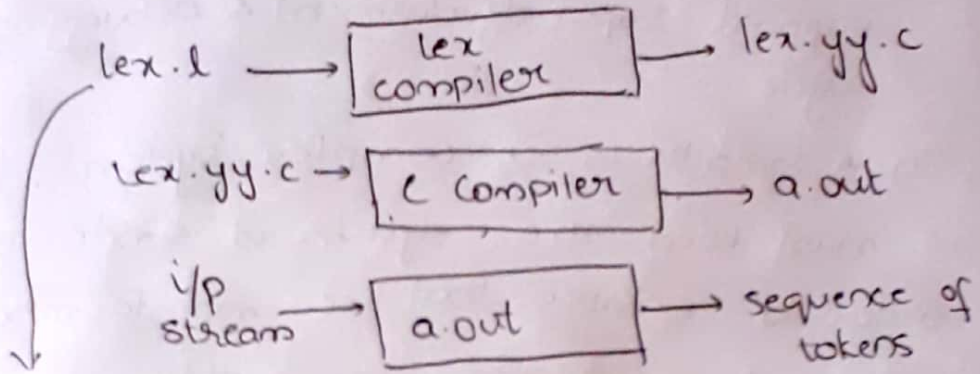
write down all the combinations where is you

put ✓ in the table.

$(q_0, q_1)$   $(q_0, q_3)$   $(q_0, q_5) \rightarrow (q_0, q_4)$



Design of lexical Analyzer Generator :-





## Science in building of a compiler :-

• Steps that have to be undertaken in order to design a compiler are :-

(1) The optimization must be correct i.e., in order ~~the~~ to preserve the meaning of a compiler program

(2) The optimization must include and improve the performance of many i/p programs

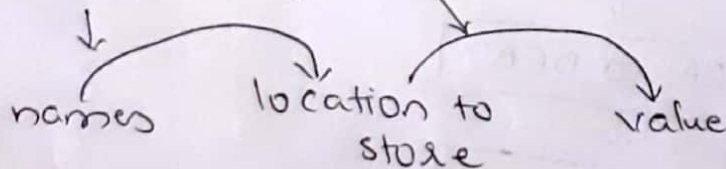
(3) The compilation time must be short to support rapid development & debugging cycle

(4) A compiler is a complex system, so, we must keep the system as simple as possible in order to ensure that the maintenance & engineering cost are manageable

## Programming languages

(1) We need to specify if we are using the static/dynamic representations

(2) Environment & states

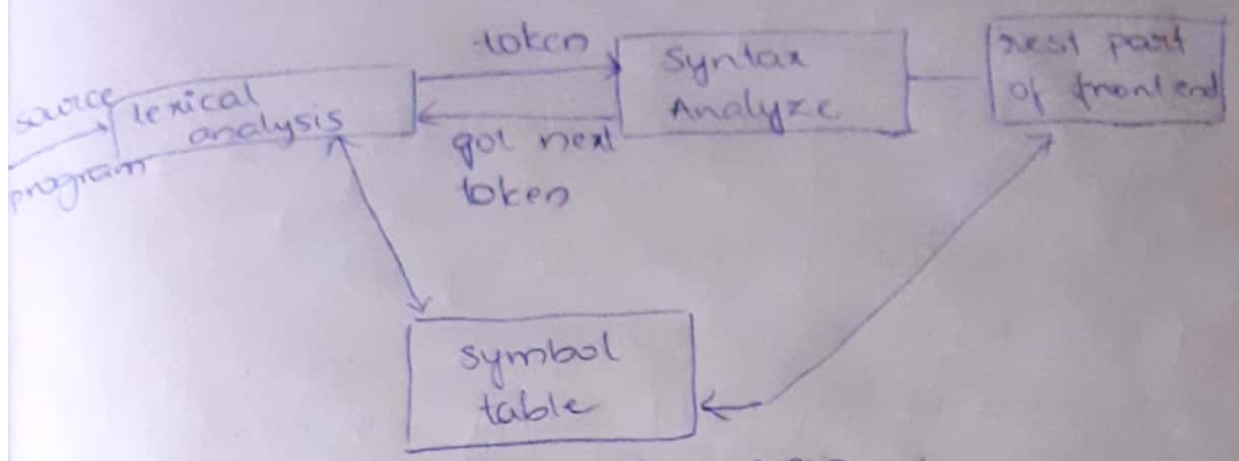


(3) Block structure :- It shd to be answerable to public & private data

(4) Parameter passing : call by reference, call by value

(5) Aliasing :- Changes done to one object is reflected on another object

# Syntax Analysis :



There are 3 techniques of parsing :-

- (1) universal parsing & techniques - It is inefficient to solve parsing
- (2) Top down parsing
- (3) Bottom up parsing

~~universal parsing~~  
 universal parsing technique — [
 

- cocke younger kasami Alg
- Earley's Alg

Grammar → set of rules  
 language → set of strings

Notational Conventions :- It is used by parsing techniques

## (1) Terminals :-

→ Terminals can be represented by

early lowercase alphabets : a, b, c, d...

operator symbols : +, -, \*, /, ^...

Punctuational symbols :- (, ), {, }, ;, ,

The digits - 0, 1, 2... 9

Bold strings - if else etc.

## (2) Non terminals :-

These symbols are non-terminals  
early uppercase alphabets - A, B, C

lower case italic strings - expr, stmt

The start symbol :- S

3) The late uppercase alphabets, suppose <sup>X, Y, Z</sup>  
are both terminals & non terminals.

These are called Grammar symbols

4) late lower case alphabets, suppose x, y, z  
are used to represent string with terminals

5) lower-case greek letters  $\alpha, \beta, \gamma$  are used  
to represent a string with terminals and  
non-terminals (variables)

6)  $A \rightarrow \alpha_1, A \rightarrow \alpha_2, A \rightarrow \alpha_3 \dots A \rightarrow \alpha_n$  are  
a set of productions. Left hand side non terminal  
is common and then it can be written as

$$A \rightarrow \alpha_1 | \alpha_2 | \alpha_3 | \dots | \alpha_n$$

$\alpha_1, \alpha_2, \alpha_3 \dots \alpha_n$  are alternatives of A.

7) unless otherwise stated the left side of  
first production is the start symbol.

Type(2) or contextfree Grammar (CFG):

CFG is accepted by pushdown Automata

A Grammar  $G = (V, T, P, S)$  is said to be  
type 2 or CFG if all the productions are  
of the form  $A \rightarrow \alpha$  where  $\alpha \in (V \cup T)^*$   
and A is a single non terminal

The symbol  $\epsilon$  can appear on right hand  
side of any production.



language generated by this grammar is called as type-2 or context free grammar

$$\begin{array}{l} A \rightarrow \emptyset \checkmark \\ B \rightarrow \epsilon \checkmark \end{array} \quad \begin{array}{l} B \rightarrow Aab/E \checkmark \\ Aa \rightarrow B \times \end{array} \quad \begin{array}{l} AE \rightarrow Ba \times \end{array}$$

Elimination of ambiguity :-

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid E \uparrow E \mid - E \mid (E) \mid id$$

The above production is ambiguous. we can dis-ambiguate the above grammar by specifying associativity and precedences

- (i) ( )
- (ii) - (unary minus)
- (iii)  $\uparrow$
- (iv) \*, /
- (v) +, -

we introduce a non terminal for each precedence level

$$\Rightarrow \text{element} \rightarrow (exp) / id$$

A subexpression that is essentially undivisible we call as an element. Element is a single identifier or (exp)

↓  
parenthesized expression.

→ we introduce a set of primaries which are the elements with 0 or more occurrences of the operations of highest precedence i.e. - unary minus

primary  $\rightarrow$  - primary / element

→ Then we construct factors as a sequence of one or more primary connected by exponential operator.

factor  $\rightarrow$  primary  $\uparrow$  factor / primary

→ Next we introduce term which are the sequence of one or more factors connected by multiplication & division operator.

term  $\rightarrow$  term \* factor / term / factor / term

→ Next we introduce expression which are the sequences of one or more terms connected by + or - operators

exp  $\rightarrow$  exp + term / exp - term / term

→ E  $\rightarrow$  E + T / E - T / T

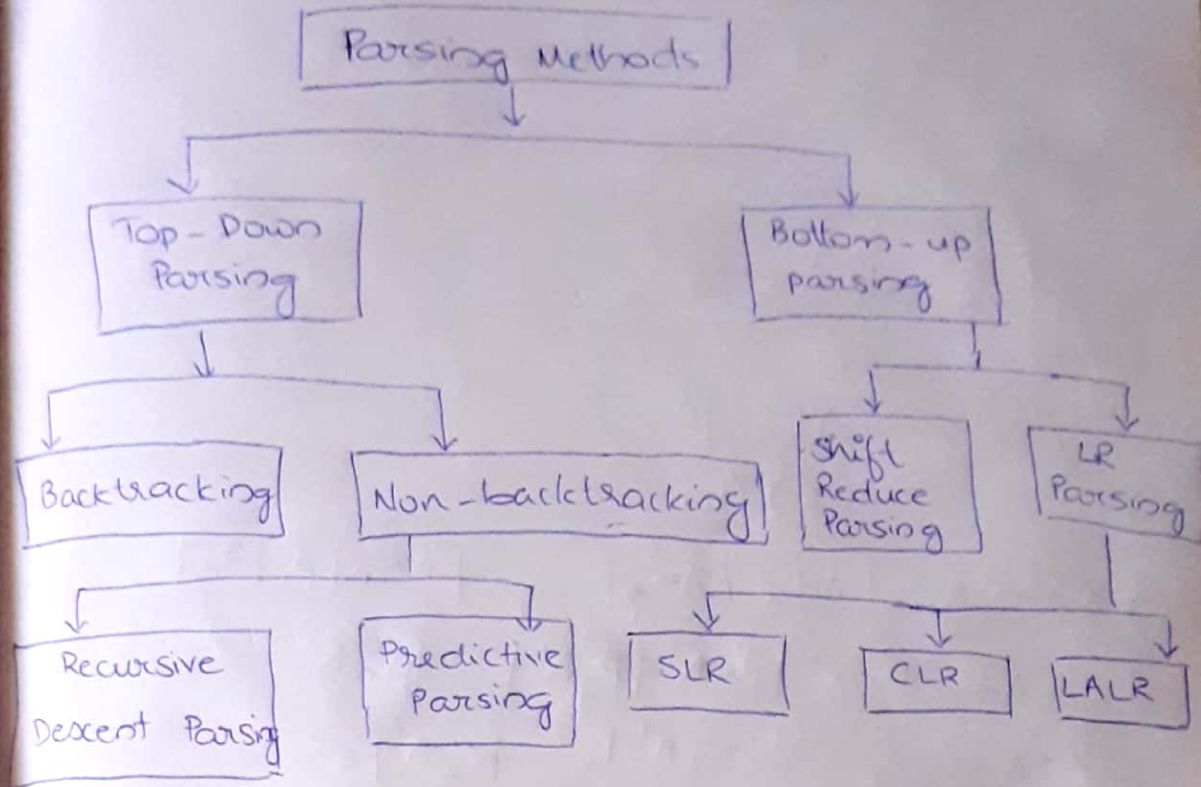
T  $\rightarrow$  T \* F / T / F / F

F  $\rightarrow$  P  $\uparrow$  f / P

P  $\rightarrow$  - P / A

A  $\rightarrow$  ( E ) / id

# Classification of parsing techniques:



## Left Recursion:-

- If a production is of form  $A \rightarrow AX/B$  where  $A$  is a non-terminal present on left hand side of the production, then the grammar is left recursive.
- Top-Down parsing techniques cannot handle left recursive grammar. So, the transformation that eliminates left recursion is needed.

The production  $A \rightarrow AX/B$  could be replaced with

$$A \rightarrow \beta A'$$

$$A' \rightarrow \alpha A' / \epsilon$$

examples :

$$1) E \rightarrow E+T/T$$

$$T \rightarrow T * F / F$$

$$F = (E) / d$$

$$E \rightarrow E+T/T \quad E \rightarrow TE'$$

$$E' \rightarrow TE' / \epsilon$$

$$T \rightarrow T * F / F$$



$$T \rightarrow FT'$$

$$T' \rightarrow *FT' / \epsilon$$

solution:

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' / \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' / \epsilon$$

$$F \rightarrow (\epsilon) / id$$

$$2) A \rightarrow A\alpha_1 / A\alpha_2 / A\alpha_3 / \dots / A\alpha_n / \beta_1 / \beta_2 / \dots / \beta_m$$

$$A \rightarrow \beta_1 A' / \beta_2 A' / \beta_3 A' / \dots / \beta_m A'$$

$$A' \rightarrow \alpha_1 A' / \alpha_2 A' / \dots / \alpha_n A' / \epsilon$$

consider an example,

$$1) S \rightarrow Aa|b$$

$$A \rightarrow Ac | Sd / \epsilon$$

$$A \rightarrow Ac / Aad / bdl / \epsilon$$

$$\underbrace{A\alpha_1} / A\alpha_2 / \beta_1 / \beta_2$$

$$A \rightarrow \beta_1 A' / \beta_2 A' \Rightarrow A \rightarrow bda' / \epsilon A'$$

$$A' \rightarrow CA' / AadA' / \epsilon$$

left factoring :-

→ If  $A \rightarrow \alpha\beta_1 / \alpha\beta_2$  are the 2 A productions, then and the input begins with a non-empty string derived from  $\alpha$ , where we do not know whether to expand  $\alpha\beta_1 / \alpha\beta_2$ .

→ so we replace these productions by

$$A \rightarrow \alpha A'$$

where  $A' \rightarrow \beta_1 / \beta_2$

Consider an example :

$$S \rightarrow iEtS \mid iEtSes/a$$

$$E \rightarrow b$$

solution :

$$S \rightarrow iEtSS'/a$$

$$S' \rightarrow E/es \Rightarrow S' \rightarrow es/E$$

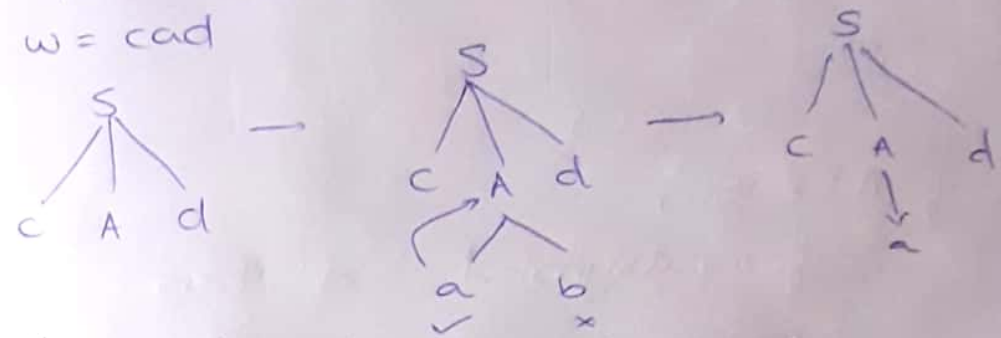
$$E \rightarrow b$$

Backtracking :-

$$S \rightarrow cAd$$

$$A \rightarrow ab/a$$

$$w = cad$$



while parsing of an input string, if an alternative of a non-terminal does not matches, with the symbols of parsing then we need to go back to the non-terminal and choose the other alternative of that non-terminal in order to parse the ip string. the technique is called as backtracking.

- choosing of alternative is disadvantage of backtracking
- backtracking itself
- left recursion

Disadvantages of top down parsing :-

- Backtracking
- left recursion

- order of alternatives
- report of failures

### Recursive Procedures :-

(i) Recursive procedures used in backtracking technique of top down parsing.

(a) procedure S() ( $S \rightarrow cad$ )

procedure S()

begin

if input symbol = 'c' then

begin

ADVANCE()

if A() then

if input symbol = 'd' then

begin

ADVANCE()

return true;

end;

end;

return false;

(b) Procedure A()

procedure A()

begin

1. save = input pointer;

if input symbol = 'a' then

begin

ADVANCE()

if input symbol = 'b' then

begin

ADVANCE()



```

returns true;
end;
end;
input pointer = i save
if input symbol = 'a' then
begin
  ADVANCE()
  returns true;
end;
returns false;
end;

```

Recursive descent parsing :-

Consider the example in order to represent non-backtracking recursive descent parsing

$$E \rightarrow E+T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

After eliminating left recursion

New productions are :-

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (E) \mid id$$

Note :- Recursive descent parsing makes use of following control structures which tells us which alternative is the only one that would be possibly succeed if we are to find a statement

stmt  $\rightarrow$  if condition then stmt else stmt  
while condition do stmt

begin statement - list end

$\rightarrow$  Recursive procedures for non terminals of  
above grammar used in recursive descent parsing

procedure E():-

```
begin
  T();
  Eprime();
```

end

procedure Eprime();

```
if input symbol = '+' then
  begin
    ADVANCE();
    T();
    EPRIME();
  end;
```

procedure T();

```
begin
  F();
  TPRIME();
end
```

procedure TPRIME()

```
if input symbol = '*' then
  begin
    ADVANCE();
    F();
    TPRIME();
  end;
```

procedure F()

```
if input symbol = 'id' then
  ADVANCE();
```

```
elseif input symbol = '(' then
  begin
```

```

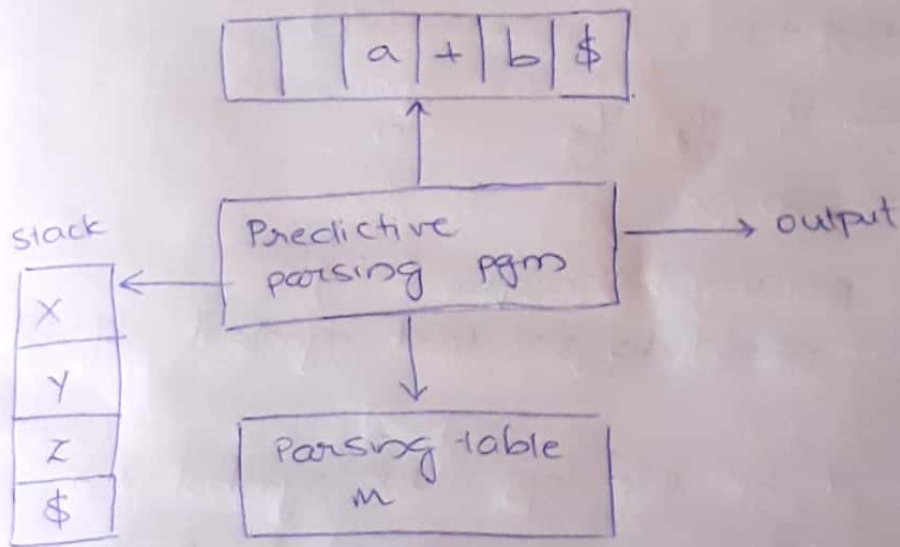
ADVANCE();
E();
if inputsymbol = ')' then
    ADVANCE();
else
    ERROR();
end;
else
    ERROR(); // print what type of error.

```

NON - recursive predictive parsing :-

The tabular implementation of recursive procedures used in recursive descent parsing is predictive parsing.

Block diagram representation of predictive parsing:



Predictive parsing technique of top-down parsing is a table given, which has an input buffer, a stack & a parsing table and an output string. The input buffer contains the string to parse, followed by \$, where \$ symbol is ~~the~~ used as right end marker to indicate, end of its string.



The stack contains the grammar symbols with  $\$$  at the bottom

- The parsing table is a 2D array indicating  $m[x, a]$  where,  $x$  is a non-terminal on the top of the stack and 'a' is a terminal or  $\$$  symbol.
- The parsing is controlled by a program that behaves as follows, where,  $x$  is a symbol on the top of the stack and 'a' is a current ip symbol then there are 3 possibilities.

(i) if  $x = a = \$$

ip string is parsed accepted

(ii) if  $x = a \neq \$$

pop  $x$  from the stack

ip++;

(iii)  $x \rightarrow Y_1, Y_2, Y_3, \dots, Y_n$

m pop  $x$

Predictive parsing technique:-

set ip to point to the first symbol of w\$,

repeat

let  $x$  be the top stack symbol & a

the symbol pointed to, by the ip

if  $x$  is terminal or  $\$$  then

if  $x = a$  then

pop  $x$  from the stack and

advance p

else

error()

else (\*x is a non terminal \*)

if  $m(x, a) = x \rightarrow y_1, y_2, \dots, y_k$  then

begin pop  $x$  from the stack ;

push  $y_k, y_{k-1}, \dots, y_3, y_2, \dots, y_1$  onto the stack with  $y$  on the top ;

output the prediction  $x \rightarrow y_1, y_2, \dots, y_k$

end

elseif

error()

Out  $x = \$$  /\* stack is empty \*/

### FIRST AND FOLLOW:

To construct predictive parsing table or to make entries to parsing table, the parser uses 2 functions FIRST AND FOLLOW

Rules to compute FIRST(x) where x is a grammar symbol:

1) if x is a terminal

$$\text{FIRST}(x) = \{x\}$$

2) if we have the predictions of the form  $x \rightarrow \epsilon$  then add  $\epsilon$  to  $\text{FIRST}(x)$

If  $x \rightarrow y_1, y_2, y_3, \dots, y_k$

then  $\text{FIRST}(x) = \text{FIRST}(y_1)$

if  $\text{FIRST}(y_1)$  contains  $\epsilon$

$$\text{then } \text{FIRST}(x) = \text{FIRST}(y_1) - \{\epsilon\} \cup \text{FIRST}(y_2)$$

again if  $\text{FIRST}(y_2)$  also contains  $\epsilon$

$$\text{then } \text{FIRST}(x) = \text{FIRST}(y_1) \cup \text{FIRST}(y_2) \cup \text{FIRST}(y_3)$$

⋮

if again  $\text{FIRST}(y_{k-1})$  also contains  $\epsilon$

$$\text{then } \text{FIRST}(x) = \text{FIRST}(y_1) \cup \text{FIRST}(y_2) \cup \dots$$

$$\text{FIRST}(\gamma_{k-1}) - \{\epsilon\} \cup \text{FIRST}(\gamma_k)$$

if  $\text{FIRST}(\gamma_k)$  also contains  $\epsilon$  then add  $\epsilon$  to  $\text{FIRST}(x)$

Rules to compute  $\text{follow}(A)$  where  $A$  non-terminal

A.

1) If  $S$  is a start symbol

$$\text{follow}(S) = \{\$ \}$$

where  $\$$  represents the right end marker of i/p.

2) If  $A \rightarrow \alpha B \beta$

$$\text{then follow}(B) = \text{FIRST}(\beta)$$

If  $\text{FIRST}(\beta)$  contains  $\epsilon$  then

$$\text{follow}(B) = \text{FIRST}(\beta) - \{\epsilon\} \cup \text{follow}(A)$$

3) If  $A \rightarrow \alpha B$

$$\text{follow}(B) = \text{follow}(A)$$

coz, there are no grammar symbols after  $B$

4) If  $X \rightarrow \gamma_1, \gamma_2, \dots, \gamma_k$  then.

$$\text{follow}(\gamma_1) = \text{FIRST}(\gamma_2)$$

If  $\text{FIRST}(\gamma_2)$  contains  $\epsilon$  then

$$\text{follow}(\gamma_1) = \text{FIRST}(\gamma_2) - \{\epsilon\} \cup \text{FIRST}(\gamma_3)$$

⋮

If  $\text{FIRST}(\gamma_k)$  also contains  $\epsilon$  then

$$\text{follow}(\gamma_1) = \text{FIRST}(\gamma_2) \cup \text{FIRST}(\gamma_3) \dots$$

$$\cup \text{FIRST}(\gamma_k) - \{\epsilon\} \cup \text{follow}(x)$$

Q for given grammar  $E \rightarrow E + T \mid T$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (\epsilon) \mid id$$

construct predictive parsing table using  $\text{FIRST}$  and  $\text{follow}$  functions and check parsing



at the i/p string  $id+id*id$   
After elimination of left recursion.

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \mid \epsilon$$

$$F \rightarrow (\epsilon)id$$

$+, -, *, /, id, (, ), \$$   
are terminals.

$$\begin{aligned} \text{FIRST}(F) &= \text{FIRST}(\epsilon) \cup \text{FIRST}(id) \\ &= \{ \epsilon, id \} \end{aligned}$$

$$\begin{aligned} \text{FIRST}(T) &= \text{FIRST}(FT') \\ &= \text{FIRST}(F) \\ &= \{ \epsilon, id \} \end{aligned}$$

$$\begin{aligned} \text{FIRST}(E) &= \text{FIRST}(TE') \\ &= \text{FIRST}(T) \\ &= \{ \epsilon, id \} \end{aligned}$$

$$\begin{aligned} \text{FIRST}(E') &= \text{FIRST}(+TE') \cup \text{FIRST}(\epsilon) \\ &= \{ +, \epsilon \} \end{aligned}$$

$$\begin{aligned} \text{FIRST}(T') &= \text{FIRST}(*FT') \cup \text{FIRST}(\epsilon) \\ &= \{ *, \epsilon \} \end{aligned}$$

After computation of  $\text{FIRST}(\epsilon)$  for all grammar symbols  $\epsilon$ . If it does not contain  $\epsilon$  then no need to compute  $\text{FOLLOW}(A)$  for all non-terminals  $A$

$$\text{FOLLOW}(E) = \{ \$, ) \}$$

$$\begin{aligned} \text{FOLLOW}(T) &= \text{FIRST}(E') \\ &= \{ +, \epsilon \}, - \{ \epsilon \} \cup \text{FOLLOW}(E) \\ &= \{ +, \$, ) \} \end{aligned}$$

$$\text{FOLLOW}(E') = \text{FOLLOW}(E) \\ = \{ \$, ) \}$$

$$\text{FOLLOW}(T') = \text{FOLLOW}(T) \\ = \{ +, \$, ) \}$$

$$\text{FOLLOW}(F) = \text{FIRST}(T') \\ = \{ *, \epsilon \} - \{ \epsilon \} \cup \text{FOLLOW}(T) \\ = \{ *, +, \$, ) \}$$

$$\text{FIRST}(E) = \{ (, \text{id} \}$$

$$\text{FIRST}(E') = \{ +, \epsilon \}$$

$$\text{FIRST}(T) = \{ (, \text{id} \}$$

$$\text{FIRST}(T') = \{ *, \epsilon \}$$

$$\text{FIRST}(F) = \{ (, \text{id} \}$$

$$\text{FOLLOW}(E) = \{ \$, ) \}$$

$$\text{FOLLOW}(E') = \{ \$, ) \}$$

$$\text{FOLLOW}(T) = \{ +, \$, ) \}$$

$$\text{FOLLOW}(T') = \{ +, \$, ) \}$$

$$\text{FOLLOW}(F) = \{ *, +, \$, ) \}$$

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' / \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' / \epsilon$$

$$F \rightarrow (E) \text{id}$$

Procedure to make entries into parsing table :

i) For each production  $A \rightarrow \alpha$  do the step 2 and 3

ii) For each terminal  $A$  present in  $\text{FIRST}(\alpha)$  add  $A \rightarrow \alpha$  to  $M[A, \alpha]$

iii) If  $\epsilon$  is present in  $\text{FIRST}(\alpha)$  then add  $A \rightarrow \alpha$  to  $M[A, b]$  so the terminal  $b$  present in  $\text{FIRST}(\alpha)$  and also add  $A \rightarrow \epsilon$  for the terminals present in  $\text{FOLLOW}(A)$ . If  $\epsilon$  is in  $\text{FIRST}(\alpha)$  and  $\$$  is in  $\text{FOLLOW}(A)$  then add  $A \rightarrow \epsilon$  to  $M[A, \$]$ .

	id	+	*	(	)	\$
E	$E \rightarrow TE'$			$E \rightarrow TE'$		
E'		$E' \rightarrow +TE'$			$E' \rightarrow \cancel{E}$	$E' \rightarrow \cancel{E}$
T	$T \rightarrow FT'$			$T \rightarrow FT'$		
T'		$T \rightarrow E$	$T' \rightarrow *TF'$		$T' \rightarrow \cancel{E}$	$T' \rightarrow \cancel{E}$
F	$F \rightarrow id$			$F \rightarrow (E)$		

NOTE :

(i) If a variable is at the right end <sup>of a production</sup>,  $\therefore \text{follow}(E) = \text{follow}(\text{left hand side})$ .

(ii) If a variable comes to the right end by substituting its right hand side variables with  $\epsilon$ , then  $\text{follow}(\text{variable}) = \text{follow}(\text{left hand side})$ .

Stack	Input	Action
\$E	id+id*id\$	$E \rightarrow TE'$
\$E'T	id+id*id\$	$T \rightarrow FT'$
\$E'T'F	id+id*id\$	$F \rightarrow id$
\$E'T'id	+id*id\$	Pop id and move pointer to the next position.
\$E'T'	+id*id\$	$T' \rightarrow E$
\$E'	+id*id\$	$E' \rightarrow +TE'$
\$E'T+	+id*id\$	pop + move next
\$E'T	id*id\$	$T \rightarrow FT'$



$\$E'T'F$	$id * id \$$	$F \rightarrow id$
$\$E'T'id$	$id * id \$$	pop id, move next
$\$E'T'$	$* id \$$	$T' \rightarrow * FT'$
$\$E'T'F*$	$* id \$$	pop *, move next
$\$E'T'F$	$id \$$	$F \rightarrow id$
$\$E'T'id$	$id \$$	pop id, move next
$\$E'T'$	$\$$	$T' \rightarrow \epsilon$
$\$E'$	$\$$	$E' \rightarrow \epsilon$
$\$$	$\$$	string is accepted.

Q. Consider the grammar

$$S \rightarrow AaAb \mid BbBa$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

construct the predictive parsing table for it and parse the input string ba

Solution:-

$$FIRST(A) = \{\epsilon\}$$

$$FIRST(B) = \{\epsilon\}$$

$$FIRST(S) = \{a, b\}$$

$$FOLLOW(A) = FIRST(aAb) \cup FIRST(b) \\ = \{a, b\}$$

$$FOLLOW(B) = FIRST(bBa) \cup FIRST(a) \\ = \{b, a\}$$

$$FOLLOW(S) = \{\$ \}$$

	a	b	⋄
S	$S \rightarrow aAb$	$S \rightarrow BbBa$	
A	$A \rightarrow \epsilon$	$A \rightarrow \epsilon$	
B	$B \rightarrow \epsilon$	$B \rightarrow \epsilon$	

stack	input	Action
\$S	ba⋄	$S \rightarrow aAb$ BbBa
\$aBbB	ba⋄	$B \rightarrow \epsilon$
\$aBb	ba⋄	pop b, move the pointer next
\$aB	a⋄	$B \rightarrow \epsilon$
\$a	a⋄	pop a, move next
\$	⋄	input string is accepted.

Consider a grammar  
 $S \rightarrow iEtSS_1 | a$   
 $S_1 \rightarrow CS | E$   
 $E \rightarrow b$

construct predictive parsing table for this grammar.

$$\text{FIRST}(S) \rightarrow \text{FIRST}(iEtSS_1) \cup \text{FIRST}(a)$$

$$= \{i, a\}$$

$$\text{FIRST}(S_1) \rightarrow \text{FIRST}(CS) \cup \text{FIRST}(E)$$

$$= \{c, \epsilon\}$$

$$\text{FIRST}(E) = \{b\}$$

$$\text{FOLLOW}(S) = \{\$ \}$$

$$\text{FOLLOW}(S_1) = \{\$ \} \cup \text{FIRST}(S_1)$$

$$= \{\epsilon, \$ \}$$

$$= \{\$, c, \epsilon\} - \{\epsilon\} \cup \text{FOLLOW}(S)$$

$$\text{FOLLOW}(S) = \{ \$, c \}$$

$$\text{FOLLOW}(S_1) = \text{FOLLOW}(S) \\ = \{ c, \$ \}$$

$$\text{FOLLOW}(E) = \{ t \}$$

Predictive parsing table :

	i	t	c	a	b	\$
S	$S \rightarrow iEtSS_1$			$S \rightarrow a$		
$S_1$			$S_1 \rightarrow cS$ $S_1 \rightarrow E$			$S_1 \rightarrow E$
E					$E \rightarrow b$	

LL(1) Grammar :-

→ If the predictive parsing table does not have any multiple entries then the grammar is said to be LL(1) grammar.

→ A grammar is LL(1) if the predictive parsing table constructed for that grammar does not contain multiple entries.

FIRST L stands for left to right scanning of i/p and second L stands for leftmost derivation, and 1 in the bracket indicates that the next i/p symbol is used to decide what is to be done next in parsing process.

① Construct the predictive parsing table for the below grammar & check whether it is LL(1) or not.

$$S \rightarrow \perp AB \mid E$$

$$A \rightarrow \perp AC \mid OC$$

$$B \rightarrow OS$$

$$C \rightarrow \perp$$

Note :- Two conditions to check whether the grammar is LL(1) or not

1) for every pair of productions  $A \rightarrow \alpha \mid \beta$  if



$FIRST(\alpha) \cap FIRST(\beta) = \emptyset$  then the grammar is LL(1)

2) If the grammar is not  $\epsilon$ -free then the additional requirement is. i.e., for every pair of productions

$$A \rightarrow \alpha \mid \beta$$

If  $FIRST(\beta)$  contains  $\epsilon$  and  $FIRST(\alpha)$  doesn't contain then  $FIRST(\alpha) \cap FOLLOW(\beta) = \emptyset$

eg:-

$$S \rightarrow AaA \mid BbBa \quad A \rightarrow \epsilon \quad B \rightarrow \epsilon$$

$$FIRST(\alpha) \cap FIRST(\beta) \quad FIRST(A) = \epsilon \quad FIRST(B) = \epsilon$$

$$FIRST(\alpha) = FIRST(AaAb) \\ = \epsilon - \{\epsilon\} \cup FIRST(aAb) = \{a\}$$

$$FIRST(\beta) = FIRST(BbBa) \\ = \epsilon - \{\epsilon\} \cup FIRST(bBa) = \{b\}$$

$$a \cap b = \emptyset$$

It is LL(1)

Solution 1:

$$S \rightarrow \epsilon \mid AB \mid \epsilon$$

$$A \rightarrow \epsilon \mid AC \mid \epsilon$$

$$B \rightarrow \epsilon \mid \epsilon$$

$$C \rightarrow \epsilon \mid \epsilon$$

$$FIRST(S) = \{\epsilon, \epsilon\}$$

$$FIRST(A) = \{\epsilon, \epsilon\}$$

$$FIRST(B) = \{\epsilon\}$$

$$FIRST(C) = \{\epsilon\}$$

$$FOLLOW(S) = \{\epsilon\} \cup FOLLOW(B) \\ = \{\epsilon\}$$

$$FOLLOW(A) = FIRST(B) \cup FIRST(C) \\ = \{\epsilon, \epsilon\}$$

$$FOLLOW(B) = FOLLOW(S) \\ = \{\epsilon\}$$

$$FOLLOW(C) = \{\epsilon, \epsilon\}$$

## Predictive parsing table

	0	1	ε	\$
S		$S \rightarrow IAB$	$S \rightarrow \epsilon$	
A	$A \rightarrow OC$	$A \rightarrow IAC$		
B	$B \rightarrow OS$			
C		$C \rightarrow I$		

Thus the above grammar is LL(1) as it does not have multiple entries.

## Bottom-up parsing :-

ex:- consider a grammar

$$S \rightarrow aABe$$

$$A \rightarrow Abc \mid b$$

$$B \rightarrow d$$

Input string : abcde

check whether the i/p string is parsed or not

Solution:-

$\underline{a}bcde$   
 matched with right side production of start symbol  
 $a\underline{A}bcde$  ( $A \rightarrow b$ )  
 $aAde$  ( $A \rightarrow Abc$ )  
 $aAde$  ( $B \rightarrow d$ )  
 $aABe$   
 $S$

Reverse of rightmost derivation of starting symbol is bottom up parsing

i/p string:- abcde

$$S \rightarrow aABe$$

matched with production on right side.

$$S \xrightarrow{rm} aA\underline{B}e \quad (B \rightarrow d)$$

$$S \xrightarrow{rm} aAde \quad (A \rightarrow Abc)$$

$$S \xrightarrow{rm} a\underline{A}bcde \quad (A \rightarrow b)$$

$s \xrightarrow{rm} abcde$

Handle:- A handle of a string is a substring that matches the right side of a production and whose reduction to the non-terminal on the left side of a production represents one step along reverse of rightmost derivation. This derivation is called handle pruning.

Ex:- consider the grammar:

$E \rightarrow E + E / E * E / id$  show the handle of each right sentential form for the string  $id + id * id$

Soln:-  
 $E \rightarrow E + E$   
 $\rightarrow E + E * E$   
 $\rightarrow E + E * id$   
 $\rightarrow E + id * id$   
 $\rightarrow id + id * id$

Sentential form	Handle	Reducing production
$id + id * id$	$id$	$E \rightarrow id$
$E + id * id$	$id$	$E \rightarrow id$
$E + E * id$	$id$	$E \rightarrow id$
$E + E * E$	$E * E$	$E \rightarrow E * E$
$E + E$	$E + E$	$E \rightarrow E + E$
$E$		

Consider the following grammar  $S \rightarrow (L) | a$   
 $L \rightarrow L, S | \epsilon$ . Show that the handle of each right sentential form for the string  $(a, (a, a))$

$S \rightarrow (L)$   
 $S \rightarrow (L, S) \quad L \rightarrow L, S$   
 $S \rightarrow (L, (L)) \quad S \rightarrow (L)$   
 $S \rightarrow (L, (L, S)) \quad L \rightarrow L, S$   
 $S \rightarrow (L, (L, a)) \quad S \rightarrow a$



$S \rightarrow (L, (S, a))$        $L \rightarrow S$

$S \rightarrow (L, (a, a))$        $S \rightarrow a$

$S \rightarrow (S, (a, a))$        ~~$S \rightarrow a$~~        $L \rightarrow S$

$S \rightarrow (a, (a, a))$        $S \rightarrow a$

Sentential form	handle	Reducing production
$(a, (a, a))$	$a$	$S \rightarrow a$
$(L, (a, a))$	$S$	$L \rightarrow S$
$(L, (L, a))$	$a$	$S \rightarrow a$
$(L, (S, a))$	$S$	$L \rightarrow S$
$(L, (L, L))$	$a$	$S \rightarrow a$
$(L, (L, S))$	$\{L, S\}$	$L \rightarrow L, S$
<del><math>(L, (L, L, L))</math></del>		
$(L, (L))$	$(L)$	$S \rightarrow (L)$
$(L, S)$	$\{L, S\}$	$L \rightarrow L, S$
$(L)$	$(L)$	$S \rightarrow (L)$
$S$		

Shift reduce parsing :- This technique is convenient way to implement bottom up parsing of an i/p string where it consists of a stack which is empty with  $\$$  symbol at bottom. An i/p buffer consisting of  $w\$$  where  $w$  is an i/p string.

The parser operates by shifting 0 or more i/p symbols onto the stack until a handle  $\beta$  is on the top of the stack.

The 4 actions performed by shift reduce parser are :-

i) In shift action, the next i/p symbol is shifted on the top of the stack.

(ii) In reduce action, the parser knows the right end of the handle is at top of the stack. It must take or locate the left end of the handle within the stack and beside with what non-terminal to replace the handle.

(iii) In an accept action, the parser announces the successful completion parsing of i/p string

(iv) In an error action, the parser discovers the syntax error has occurred and cause error recovery routine.

ex:- For the given grammar  $S \rightarrow (L) | a$   
 $L \rightarrow \epsilon | L, S | \wedge$

using shift reduce parsing parse the i/p string  $(a, (a, a))$

soln:-

$S \rightarrow (L) | a$

$L \rightarrow L, S | \wedge$

stack	Input buffer	Reduced production
\$	$(a, (a, a)) \$$	shift (
\$(	$a, (a, a)) \$$	shift a
\$(a	$, (a, a)) \$$	Reduce $s \rightarrow a$
\$(s	$, (a, a)) \$$	Reduce $L \rightarrow s$
\$(L	$, (a, a)) \$$	shift ,
\$(L,	$(a, a)) \$$	shift (
\$(L,(	$a, a)) \$$	shift a
\$(L,(a	$, a)) \$$	Reduce $s \rightarrow a$
\$(L,(s	$, a)) \$$	Reduce $L \rightarrow s$
\$(L,(L	$, a)) \$$	shift ,
\$(L,(L,	$a)) \$$	shift a

$\$(L, (L, a))$

$\$(L, (L, S$

$\$(L, (L$

$\$(L, (L)$

~~$\$(L, S)$~~

$\$(L, S)$

$\$(L$

$\$(L)$

$\$S$

)\$

)\$

)\$

)\$

~~)\$~~

)\$

)\$

\$

\$

Reduce  $S \rightarrow (L)$   
 Reduce  $L \rightarrow L, S$   
 Shift  $($   
 Reduce  $S \rightarrow (L)$   
 Shift  $)$   
 Reduce  $L \rightarrow L, S$   
 Shift  $)$   
 Reduce  $S \rightarrow (L)$

5. Parse i/p string  $id + id * id$  for the grammar.

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

using  
Shift reduce parsing

6. Parse i/p string  $int \ id, id;$  using shift reduce parsing for the grammar.

$$S \rightarrow TL$$

$$T \rightarrow int \mid float$$

$$L \rightarrow L, id \mid id$$

Operator precedence grammar :-

Operator grammar :- The grammar which has no  $\epsilon$  on the right side of the production and no 2 adjacent non terminals is called operator grammar.   
  $\rightarrow$  Two adjacent non-terminals

ex:-  $E \rightarrow EAE \mid (E) \mid -E \mid id$

This is not an operator grammar

$$A \rightarrow + \mid - \mid * \mid / \mid \uparrow$$

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid \uparrow E \mid (E) \mid -E \mid id$$

This is an operator grammar.



Operator precedence grammar makes use of 3 disjoint precedence relation b/w certain pair of ~~terms~~ terminals

i)  $a < \cdot b$  " a yields precedence to b "

ii)  $a \dot{=} b$  " a having same precedence to b "

iii)  $a \cdot > b$  " a having precedence over b "

Ex: Consider operator precedence relations for the right sentinal form  $id + id * id$

	id	+	*	\$
id		$\cdot >$	$\cdot >$	$\cdot >$
+	$< \cdot$	$\cdot >$	$< \cdot$	$\cdot >$
*	$< \cdot$	$\cdot >$	$\cdot >$	$\cdot >$
\$	$< \cdot$	$< \cdot$	$< \cdot$	

using precedence table the input  $id + id * id$  is written as:

$\$ < \cdot id \cdot > + < \cdot id \cdot > * < \cdot id \cdot >$

$< \cdot id \cdot >$  — it is called as handle.

Steps undertaken by operator precedence grammar in order to identify handle.

→ scan ~~if~~ string from left end until the first  
 i) precedence is found then scan backwards  
 over  $=$  until  $< \cdot$  is found.

→ The left of  $\cdot >$  and right of  $< \cdot$  is considered  
 as handle

operator precedence relation based upon  
 precedences and associativity

i) If operator  $\theta_1$  has higher precedence than  
 operator  $\theta_2$  then,  $\theta_1 \cdot > \theta_2$  and  $\theta_2 < \cdot \theta_1$

i) If operator  $\theta_1$  has the same precedence with operator  $\theta_2$  then,

$\theta_1 \cdot \rightarrow \theta_2$  and  $\theta_2 \cdot \rightarrow \theta_1$ , if operator is left associative  
(+, -)

$\theta_1 < \cdot \theta_2$  and  $\theta_2 < \cdot \theta_1$ , if operator is right associative (\*, /)

(iii) If  $\theta$  is any operator, then make

$\theta < \cdot id$

$id \cdot \rightarrow \theta$  (because id has highest precedence over all)

$\$ < \cdot \theta$       \$ has the least precedence.

$\theta \cdot \rightarrow \$$        $\theta$  has higher precedence.

$\theta < \cdot ( \quad ) \cdot \rightarrow \theta$

$( < \cdot \theta \quad \theta \cdot \rightarrow )$

apart from these, also make  $( \equiv )$ ,  $( < \cdot ( , ) \cdot \rightarrow )$

$id \cdot \rightarrow \$$        $( < \cdot id$

$\$ < \cdot ($   ~~$\$ < \cdot id$~~

$\$ < \cdot id$

$id \cdot \rightarrow )$

$( \cdot \rightarrow \$$

Based upon the above rules, construct the operator precedence relation for the above string

id\*(id↑id)-id/id

+, -, \* left associative, ↑, id, (, ) \$ right associative

	+	-	*	↑	id	(	)	\$	
+	·>	·>	<·	<·	<·	<·	<·	·>	·>
-	·>	·>	<·	<·	<·	<·	<·	·>	·>
*	·>	·>	·>	·>	<·	<·	<·	·>	·>
/	·>	·>	·>	·>	<·	<·	<·	·>	·>
↑	·>	·>	·>	·>	<·	<·	<·	·>	·>
id	·>	·>	·>	·>	·>			·>	·>

	+	-	*	/	↑	id	(	)	\$
(	<	<	<	<	<	<	<	=	
)	>	>	>	>	>			>	>
\$	<	<	<	<	<	<	<	<	

we have to check left associative or right associativity

Parsing of i/p string id + id \* id

stack

\$

\$ < . id

\$ < . id . >

\$ < . + < .

\$ < . + < . id . >

~~\$ < . + < .~~

\$ < . + < . id . >

\$ < . + < . id . >

input buffer

id + id \* id \$

+ id \* id \$

+ id \* id \$

id \* id \$

\* id \$

id \$

\$

\$

Advantages:-

- operator precedence technique is easy to implement

Disadvantage:-

- only small class of grammar are supported
- It can't parse/assurance of acceptance of i/p string for desired language
- It will not support or handle operators which is having 2 different precedences.



Precedence function:-

we make use of 2 functions  $f$  and  $g$ , then  
for symbols  $A$  &  $B$  we can write

i) if  $f(a) < g(b)$  whenever  $a < b$

ii) if  $f(a) > g(b)$  whenever  $a > b$

iii)  $f(a) = g(b)$  whenever  $a = b$

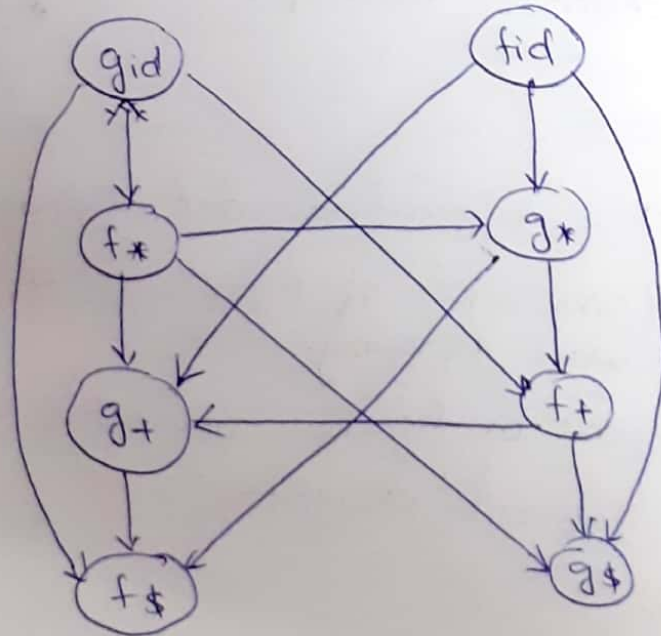
Consider an example which represents the  
precedence matrix for the terminals  $id, +, *, \$$

		$g$			
		$id$	$+$	$*$	$\$$
$id$			$\cdot \rightarrow$	$\cdot \rightarrow$	$\cdot \rightarrow$
$+$	$+$	$< \cdot$	$\cdot \rightarrow$	$< \cdot$	$\cdot \rightarrow$
$*$	$*$	$< \cdot$	$\cdot \rightarrow$	$\cdot \rightarrow$	$\cdot \rightarrow$
$\$$	$\$$	$< \cdot$	$< \cdot$	$< \cdot$	

$id, +, *, \$$  has no equal precedence so  
draw separate nodes using  $f$  and  $g$

if  $f(a) \cdot \rightarrow g(b)$  draw  $f(a) \rightarrow g(b)$

if  $g f(a) < \cdot g(b)$  draw  $f(a) \leftarrow g(b)$



	+	*	id	\$
f	2	3	4	0
g	1	3	5	0

Q. Consider a precedence matrix for +, \*, (, ), id, \$

	+	*	(	)	id	\$
+	>	<	<	>	<	>
*	>	>	<	>	<	>
(	<	<	<	=	<	
)	>	>	=	>		>
id	>	>	>	>		>
\$	<	<	<		<	

For this matrix, draw precedence graph & also mention the numerical values for function f and g w.r.t all terminals.

LR Parsing:-

Precedence Functions :-

Compilers using operator precedence parsers need not store the table of precedence relations. In most cases, the table can be encoded by the two precedence functions f & g that map terminal symbols to integers. We attempt to select f & g so that for symbols a and b.

- 1)  $f(a) < g(b)$  whenever  $a < b$
- 2)  $f(a) = g(b)$  whenever  $a = b$
- 3)  $f(a) > g(b)$  whenever  $a > b$

The precedence relation b/w a & b can be determined by a numerical comparison b/w

$f(a)$  and  $g(b)$

Eg:- The precedence functions for the precedence table consisting of  $+, -, *, |, \uparrow, (, ), id, \$$  is

	$+$	$-$	$*$	$ $	$\uparrow$	$($	$)$	$id$	$\$$
$f$									0
$g$									0

Algorithm : Constructing precedence functions:-

Input :- An operator precedence matrix

Output :- Precedence functions representing the  $\uparrow P$  matrix or an indication that none exists

Method:-

- 1) Create symbols  $f_a$  &  $g_b$  for each  $a$  or  $b$   
 $a \rightarrow$  i.e terminal or  $\$$
- 2) partition the created symbols into as many groups as possible in such a way that if  $a = b$ , then  $f_a$  &  $g_b$  are in the same group
- 3) create a directed graph whose nodes are the groups in ②  
for any  $a$  &  $b$ , if  $a < b$ ,  $P$



construct SLR parsing table for the following grammar.

$$E \rightarrow E+T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

Step 1 :- Augment the grammar & add a  $\cdot$  operator at the beginning of every grammar.

$$E' \rightarrow \cdot E$$

$$E \rightarrow \cdot E+T$$

$$E \rightarrow \cdot T$$

$$T \rightarrow \cdot T * F$$

$$T \rightarrow \cdot F$$

$$F \rightarrow \cdot (E)$$

$$F \rightarrow \cdot id$$

( $\therefore$  Always begins with

$$S' \rightarrow \cdot S$$

Start symbol)

Step 2: find closure ( $E' \rightarrow E$ ) :  $I_0$

$$E' \rightarrow \cdot E$$

$$E \rightarrow \cdot E+T$$

$$E \rightarrow \cdot T$$

$$T \rightarrow \cdot T * F$$

$$T \rightarrow \cdot F$$

$$F \rightarrow \cdot (E)$$

$$F \rightarrow \cdot id$$

shift the  $\cdot$  to the next place.

Step 3 :- closure [goto( $I_0, E$ )]

$$E' \rightarrow E \cdot$$

$$E \rightarrow E \cdot +T$$

$I_1$

nonterminals /  
grammar  
symbols  
terminals

Step 4: closure [goto( $I_0, T$ )]

$$T \rightarrow T \cdot * F$$

$$T \rightarrow T \cdot F$$

$$E \rightarrow T \cdot$$

$I_2$

step 5 :- closure [goto [I<sub>0</sub>, F]]  
T → F. } I<sub>3</sub>

step 6 :- closure [goto [I<sub>0</sub>, (]]  
closure { F → (.E) }  
E → .E + T  
E → .T  
T → .T \* F  
T → .F  
F → .(E)  
F → .id } I<sub>4</sub>

step 7 :- closure [goto [I<sub>0</sub>, id]]  
F → id. } I<sub>5</sub>

Step 8 :- perform closure [goto [I<sub>i</sub>, non-terminal/grammatical symbol/terminals]]

(i) closure [goto [I<sub>1</sub>, +]] → I<sub>6</sub>  
closure { E → E + .T }

using T from I<sub>0</sub>

T → .T \* F  
T → .F  
F → .(E)  
F → .id

(ii) closure [goto [I<sub>2</sub>, \*]] → I<sub>7</sub>

closure { T → T \* .F }  
F → .(E)  
F → .id

(iii) closure [goto [I<sub>4</sub>, E]]

F → (E.) } I<sub>8</sub> ✓  
E → E + T

(iv) closure [goto [I<sub>4</sub>, T]] → already exists

$$\left. \begin{array}{l} E \rightarrow T. \\ T \rightarrow T.*F \end{array} \right\} I_2$$

(v) closure of  $[goto(I_4, F)]$   
 $T \rightarrow F. \quad \} I_3$

(vi) closure  $[goto(I_4, ( ])$   
 closure  $\{ F \rightarrow (.E)$   
 $E \rightarrow .E+T$   
 $E \rightarrow .T$   
 $T \rightarrow .T*F$   
 $T \rightarrow .F$   
 $F \rightarrow .(E)$   
 $F \rightarrow .id$

closure  $[goto(I_4, id)] = I_5$

(vii) closure  $[goto(I_6, T)]$   
 $E \rightarrow E+T.$   
 $T \rightarrow T.*F$

(viii) closure  $[goto(I_6, F)] = I_3$   
 closure  $[goto(I_6, ( ] = I_4$   
 closure  $[goto(I_6, id)] = I_5$

(ix) closure  $[goto(I_7, F)] = I_{10}$   
 $T \rightarrow T.*F.$

(x) closure  $[goto(I_7, ( ] = I_4$   
 closure  $[goto(I_7, id)] = I_5$

(xi) closure  $[goto(I_8, )]] = I_{11}$   
 $F \rightarrow (E).$

closure  $[goto(I_8, +)] = I_6$

(xii) closure  $[goto(I_9, *)] = I_7$   
 closure  $[goto(I_9,$



$$\text{closure}(\text{goto}(I_0, E)) = I_1$$

$$\text{closure}(\text{goto}(I_0, T)) = I_2$$

$$\text{closure}(\text{goto}(I_0, F)) = I_3$$

$$\text{closure}(\text{goto}(I_0, ( ))) = I_4$$

$$\text{closure}(\text{goto}(I_0, id)) = I_5$$

$$\text{closure}(\text{goto}(I_1, +)) = I_6$$

$$\text{closure}(\text{goto}(I_2, *)) = I_7$$

$$\text{closure}(\text{goto}(I_4, E)) = I_8$$

$$\text{closure}(\text{goto}(I_4, T)) = I_2$$

$$\text{closure}(\text{goto}(I_4, F)) = I_3$$

$$\text{closure}(\text{goto}(I_4, ( ))) = I_4$$

$$\text{closure}(\text{goto}(I_6, T)) = I_9$$

$$\text{closure}(\text{goto}(I_4, id)) = I_5$$

$$\text{closure}(\text{goto}(I_6, F)) = I_3$$

$$\text{closure}(\text{goto}(I_6, ( ))) = I_4$$

$$\text{closure}(\text{goto}(I_6, id)) = I_5$$

$$\text{closure}(\text{goto}(I_7, F)) = I_{10}$$

$$\text{closure}(\text{goto}(I_7, ( ))) = I_4$$

$$\text{closure}(\text{goto}(I_7, id)) = I_5$$

$$\text{closure}(\text{goto}(I_8, ( ))) = I_{11}$$

$$\text{closure}(\text{goto}(I_8, +)) = I_6$$

$$\text{closure}(\text{goto}(I_9, *)) = I_7$$

state	id	+	*	(	)	\$	E	T	F
0	S <sub>5</sub>			S <sub>4</sub>			1	2	3
1		S <sub>6</sub>	<del>S<sub>7</sub></del>			accepted			
2		r <sub>2</sub>	S <sub>7</sub>		r <sub>2</sub>	r <sub>2</sub>			
3		r <sub>4</sub>	r <sub>4</sub>		r <sub>4</sub>	r <sub>4</sub>			
4	S <sub>5</sub>			S <sub>4</sub>			8	2	3
5		r <sub>6</sub>	r <sub>6</sub>		r <sub>6</sub>	r <sub>6</sub>			
6	S <sub>5</sub>			S <sub>4</sub>				9	3
7	S <sub>5</sub>			S <sub>4</sub>					10
8		S <sub>6</sub>			S <sub>11</sub>				
9		r <sub>1</sub>	S <sub>7</sub>		r <sub>1</sub>	r <sub>1</sub>			
10		r <sub>3</sub>	r <sub>3</sub>		r <sub>3</sub>	r <sub>3</sub>			
11		r <sub>5</sub>	r <sub>5</sub>		r <sub>5</sub>	r <sub>5</sub>			

$$\text{FIRST}(E) = \{ \text{id}, ( \}$$

$$\text{FIRST}(T) = \{ \text{id}, ( \}$$

$$\text{FIRST}(F) = \{ \text{id}, ( \}$$

$$\text{FOLLOW}(E) = \{ \$, +, ) \}$$

$$\text{FOLLOW}(T) = \{ *, \$, +, ) \}$$

$$\text{FOLLOW}(F) = \{ *, \$, +, ) \}$$

$$I_1, I_2, I_3, I_5, I_9, I_{10}, I_{11}$$

→ choose the states that contain a production that ends with a .

$$1. E \rightarrow E + T$$

$$2. E \rightarrow T$$

$$3. T \rightarrow T * F$$

$$4. T \rightarrow F$$

$$5. F \rightarrow (E)$$

$$6. F \rightarrow \text{id}$$

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' / \epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' / \epsilon$$

$$F \rightarrow (E) \text{id}$$

• The states that contain the production that 'end' with .

Steps.

1) In state  $I_1$ , the production that ends with . is  $E' \rightarrow E$ . Since it is an augment production make the entry as "accept" and with the state  $I_1$  and the terminal  $\$$

2) In state  $I_2$ , the production that ends with . is  $E \rightarrow T$ . So in the given grammar the production number is 2, so make the entry as  $r_2$ . In state  $I_2$  w.r.t. all the terminals in FOLLOW(E)

$E \rightarrow T$  is  $r_2$  fill it in FOLLOW(E)

3)  $I_3$   $T \rightarrow F$ . is production  $r_4$

fill the table with FOLLOW(T)

4)  $I_5$ :  $F \rightarrow id$ . is  $r_6$  fill it in table

at FOLLOW(F)

6)  $I_9$ :  $E \rightarrow E+T$ . is production  $r_1$

7)  $I_{10}$ :  $T \rightarrow T * F$ . is production  $r_3$   
fill it in FOLLOW(T)

8)  $I_{11}$ :  $F \rightarrow (E)$ . is production  $r_5$   
fill it with FOLLOW(f)



Q. with reference to the above parsing table pass the input string  $id+id*id\$$

state	i/p buffer	Action
\$0	$id+id*id\$$	Shift $s_5$
$\$oid5$	$+id*id\$$	Reduce $r_6$ $F \rightarrow id$
$\$OF3$	$+id*id\$$	Reduce $r_4$ $T \rightarrow F$
$\$OT2$	$+id*id\$$	Reduce $r_2$ $E \rightarrow T$
$\$OE1$	$+id*id\$$	Shift $s_6$
$\$OE1+6$	$id*id\$$	Shift $s_5$
$\$OE1+6id5$	$*id\$$	Reduce $r_6$ $F \rightarrow id$
$\$OE1+6F3$	$*id\$$	Reduce $r_4$ $T \rightarrow F$
$\$OE1+6T9$	$*id\$$	Shift $s_7$
$\$OE1+6T9*7$	$id\$$	Shift $s_5$
$\$OE1+6T9*7id5$	$\$$	Reduce $r_6$ $F \rightarrow id$
$\$OE1+6T9*7F10$	$\$$	Reduce $r_3$ $T \rightarrow T*F$ $\quad \quad \quad \underbrace{\quad \quad}_{3 \vee 2} = 6$
pop 6 elements		Reduce $r_1$ $E \rightarrow E+T$
$\$OE1+6T9$	$\$$	
$\$OE1$	$\$$	Accepted.

Q. With respect to the above parsing table  
 $id * id \$$  :- parse the input string

State	Input buffer	Action
$\$0$	$id * id \$$	Shift $s_5$
$\$0id5$	$* id \$$	Reduce $r_6$ $f \rightarrow id$
$\$0f3$	$* id \$$	Reduce $r_4$ $T \rightarrow f$
$\$0T2$	$* id \$$	Shift $s_7$
$\$0T2*7$	$id \$$	Shift $s_5$
$\$0T2*7id5$	$\$$	Reduce $r_6$ $f \rightarrow id$
$\$0T2*7f10$	$\$$	Reduce $r_3$ $T \rightarrow T * f$
$\$0T2$	$\$$	Reduce $r_2$ $E \rightarrow T$
$\$0E1$	$\$$	Accept.

Canonical LR Parsing technique :-

Q. Construct canonical LR Parsing table for the given grammar.

$$S \rightarrow CC$$

$$C \rightarrow aC/d$$

LR(1)

$$[A \rightarrow \alpha \cdot B\beta, a]$$

$$[B \rightarrow \gamma, 0]$$

$$[B \rightarrow \cdot \gamma, b]$$

$\uparrow$   
FIRST( $B, a$ )

Soln:-

$$S' \rightarrow \cdot S$$

$$S \rightarrow \cdot CC$$

$$C \rightarrow \cdot aC$$

$$C \rightarrow \cdot d$$

In CLR we should have the productions in the form  $[A \rightarrow \alpha \cdot B\beta, a]$

(1) closure  $[S' \rightarrow \cdot S, \$]$

compare  $[A \rightarrow \alpha \cdot B\beta, a]$

$A \rightarrow S', \alpha = \epsilon, B = S, \beta = \epsilon, a = \$$

~~closure~~  $[S \rightarrow \cdot CC, \$]$

$$\text{FIRST}(\beta a) = \text{FIRST}(\epsilon \$) \\ = \$$$

$S' \rightarrow \cdot S, \$$

closure  $[S \rightarrow \cdot CC, \$]$

$A \rightarrow \alpha \cdot B\beta, a$

$A = S, \alpha = \epsilon, B = C, \beta = C, a = \$$

$[C \rightarrow \cdot aC, a/d]$

$[C \rightarrow \cdot d, a/d]$

$$\text{FIRST}(\beta a) = \text{FIRST}(C\$) \\ = \text{FIRST}(C) \\ = \{a, d\}$$

Now write all the productions

$S' \rightarrow \cdot S, \$$   
 $S \rightarrow \cdot CC, \$$   
 $C \rightarrow \cdot aC, a/d$   
 $C \rightarrow \cdot d, a/d$  }  $I_0$

closure  $[goto(I_0, S)] = I_1$

closure  $[goto(I_0, C)] = I_2$

closure  $[goto(I_0, a)] = I_3$

closure  $[goto(I_0, d)] = I_4$

closure  $[goto(I_2, C)] = I_5$

closure  $[goto(I_2, a)] = I_6$

closure  $[goto(I_2, d)] = I_7$

closure  $[goto(I_3, C)] = I_8$

closure  $[goto(I_3, a)] = I_9$

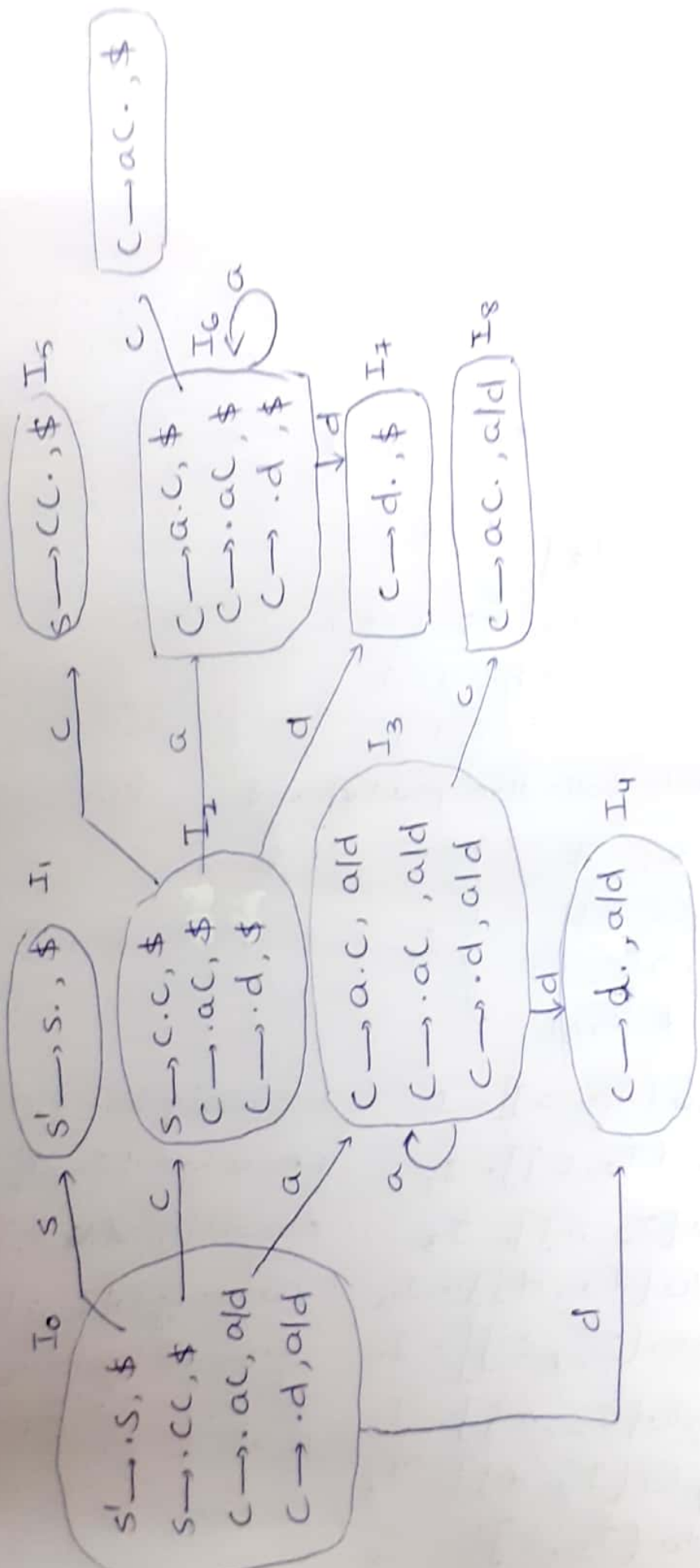
closure  $[goto(I_3, d)] = I_{10}$

closure  $[goto(I_6, C)] = I_{11}$

closure  $[goto(I_6, a)] = I_{12}$

closure  $[goto(I_6, d)] = I_{13}$





State	Action			Goto	
	a	d	\$	S	C
0	S <sub>3</sub>	S <sub>4</sub>	accept	1	2
1		S <sub>7</sub>			5
2	S <sub>6</sub>				8
3	S <sub>3</sub>	S <sub>4</sub>			
4	r <sub>3</sub>	r <sub>3</sub>			
5			r <sub>1</sub>		
6	S <sub>6</sub>	S <sub>7</sub>			9
7			r <sub>3</sub>		
8	r <sub>2</sub>	r <sub>2</sub>			
			r <sub>2</sub>		

$I_1, I_4, I_5, I_7, I_8, I_9$

1.  $S \rightarrow CC$
2.  $C \rightarrow aC$
3.  $C \rightarrow d$

For each of the states, using productions that end with a  $\cdot$ .

Place  $I_{\theta r_i}$  (where  $i$  is production number) in the look ahead terminal.

Construct w.r.t the above constructed canonical parsing table, parse the input string  $aadd$

Stack

i/p buffer

Action

\$0

$aadd\$$

Shift S<sub>3</sub>

$\$0a3$

$aadd\$$

Shift S<sub>3</sub>

$\$0a3a3$

$add\$$

Shift S<sub>4</sub>

$\$0a3a3d4$

$d\$$

Reduce

R<sub>3</sub>  $C \rightarrow d$

$\$0a3a3c8$

$cd\$$

Reduce r<sub>2</sub>  
 $C \rightarrow aC$

$\phi 0a3c8$

d\$

Reduce  $R_2$   $C \rightarrow ac$

$\phi 0c2$

d\$

Shift  $S_7$

$\phi 0c2d7$

\$

Reduce  $R_3$   
 $C \rightarrow d$

$\phi 0c2c5$

\$

Reduce  $R_1$   
 $S \rightarrow cc$

$\phi 0s1$

\$

accepted.

### LALR parsing technique :-

$I_0 : s' \rightarrow \cdot s, \$$   
 $s \rightarrow \cdot cc, \$$   
 $c \rightarrow \cdot ac, a/d$   
 $c \rightarrow \cdot d, a/d$

$I_{36} = c \rightarrow a \cdot c, a/d/\$$   
 $c \rightarrow \cdot ac, a/d/\$$   
 $c \rightarrow \cdot d, a/d/\$$

$I_1 : s' \rightarrow s \cdot, \$$

$I_{47} = c \rightarrow d \cdot, a/d/\$$

$I_2 : s \rightarrow \cdot cc, \$$   
 $c \rightarrow \cdot ac, \$$   
 $c \rightarrow \cdot d, \$$

$I_{89} = c \rightarrow ac \cdot, a/d/\$$

	a	d	\$	s	c
$I_0$	$S_{36}$	$S_{47}$		1	2
$I_1$			accept		
$I_2$	$S_{36}$	$S_{47}$			5
$I_{36}$	$S_{36}$	$S_{47}$			89
$I_{47}$	$r_3$	$r_3$	$r_3$		
$I_5$					$r_1$
$I_{89}$	$r_2$	$r_2$	$r_2$		



To generate a parse tree in LALR, we used  
YACC  $\rightarrow$  Yet Another Compiler Compiler

A grammar is said to be ambiguous, if we  
have more than 1 parse tree  
learn left parse tree & right parse tree.

Q. Construct productive parsing table for  
the grammar

$$S \rightarrow (L)/a$$

$$L \rightarrow L,S/S$$

and check if the input string  $(a,a)$  be parsed  
or not

Soln:-

$$S \rightarrow (L)/a$$

$$L \rightarrow SL'$$

$$L' \rightarrow , SL' / \epsilon$$

$$\text{FIRST}(S) = \text{FIRST}((L)) \cup \text{FIRST}(a)$$

$$= \{(, a\}$$

$$\text{FIRST}(L) = \text{FIRST}\{SL'\}$$

$$= \text{FIRST}\{S\} = \{(, a\}$$

$$\text{FIRST}(L') = \text{FIRST}\{, SL'\} \cup \text{FIRST}(\epsilon)$$

$$= \{, \epsilon\}$$

$$\text{FOLLOW}(S) = \text{FOLLOW}(L') \cup \{\$\}$$

$$= \{\$, )\}$$

$$\text{FOLLOW}(L) = \{)\}$$

$$\text{FOLLOW}(L') = \{\text{FOLLOW}(L)\} \cup \text{FOLLOW}(L')$$

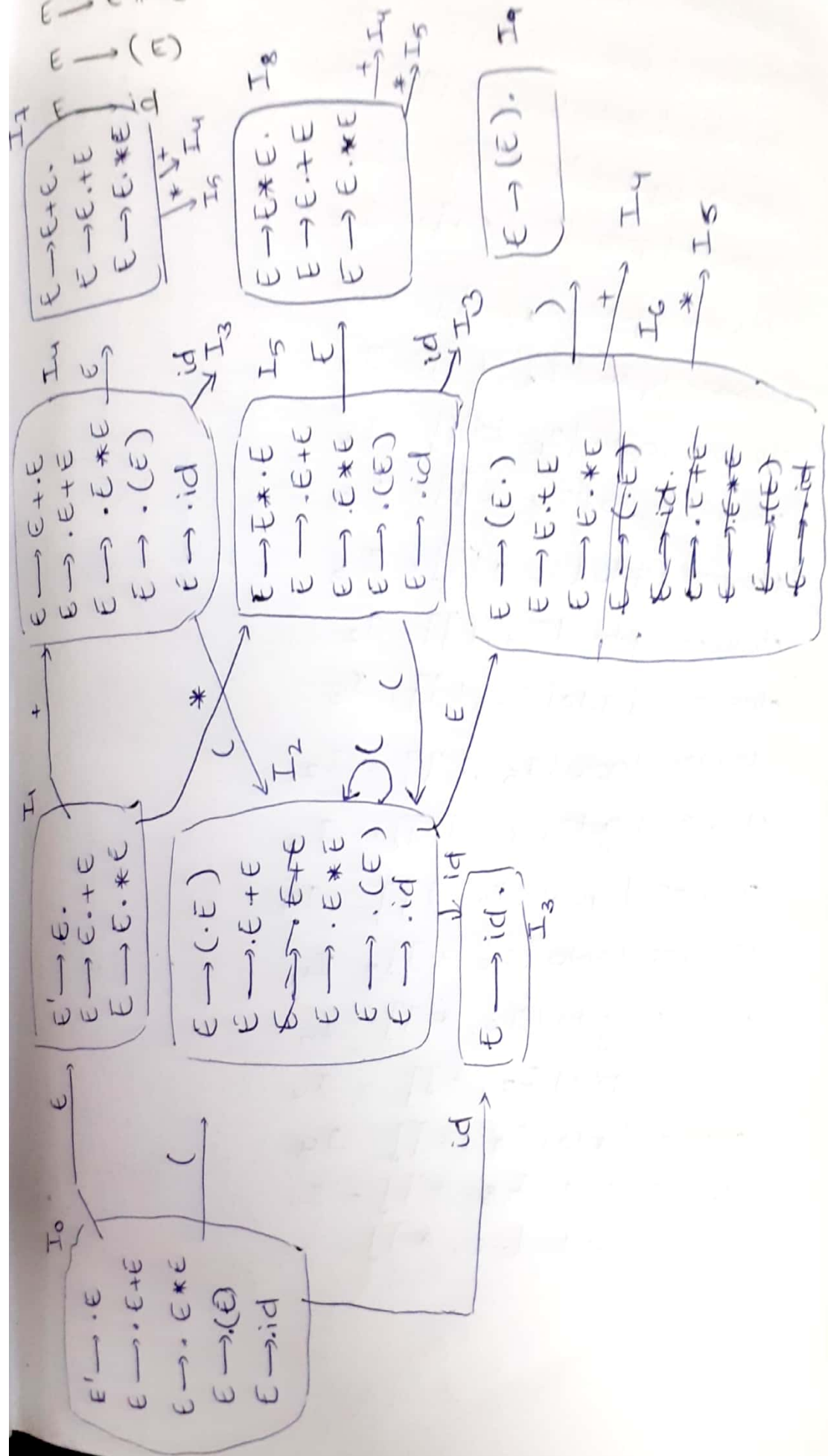
$$= \{)\}$$

	(	a	,	ε	\$	)
S	$S \rightarrow (L)$	$S \rightarrow a$				
L	$L \rightarrow SL'$	$L \rightarrow SL'$				
L'			$L' \rightarrow ,SL'$	<del>base</del>		$L' \rightarrow \epsilon$

Stack	Input	action
\$ S	(a,a)\$	$S \rightarrow (L)$
\$ )L(	(a,a)\$	pop ( and move the next pointer
\$ )L	a,a)\$	$L \rightarrow SL'$
\$ )L's	a,a)\$	$S \rightarrow a$
\$ )L'a	a,a)\$	pop a move the pointer
\$ )L'	,a)\$	$L' \rightarrow ,SL'$
\$ )L's,	,a)\$	pop , move the pointer
\$ )L's	a)\$	$S \rightarrow a$
\$ )L'a	a)\$	pop a and move the next pointer
\$ )L'	)\$	$L' \rightarrow \epsilon$
\$ )	)\$	pop ) move next pointer
\$	\$	accepted

# Handling Ambiguous Grammar

- $E \rightarrow E + E$
- $E \rightarrow E * E$
- $E \rightarrow (E)$





closure (goto (I<sub>0</sub>, ε)) = I<sub>1</sub>

closure (goto (I<sub>0</sub>, ( )) = I<sub>2</sub>

closure (goto (I<sub>0</sub>, id)) = I<sub>3</sub>

closure (goto (I<sub>1</sub>, +)) = I<sub>4</sub>

closure (goto (I<sub>1</sub>, \*)) = I<sub>5</sub>

closure (goto (I<sub>2</sub>, ε)) = I<sub>6</sub>

closure (goto (I<sub>2</sub>, ( )) = I<sub>2</sub>

closure (goto (I<sub>2</sub>, id)) = I<sub>3</sub>

closure (goto (I<sub>4</sub>, ε)) = I<sub>7</sub>

closure (goto (I<sub>4</sub>, id)) = I<sub>3</sub>

closure (goto (I<sub>4</sub>, ( )) = I<sub>2</sub>

closure (goto (I<sub>5</sub>, ε)) = I<sub>8</sub>

closure (goto (I<sub>5</sub>, ( )) = I<sub>2</sub>

closure (goto (I<sub>5</sub>, id)) = I<sub>3</sub>

closure (goto (I<sub>6</sub>, )) = I<sub>9</sub>

closure (goto (I<sub>6</sub>, +)) = I<sub>4</sub>

closure (goto (I<sub>6</sub>, \*)) = I<sub>5</sub>

closure (goto (I<sub>7</sub>, +)) = I<sub>4</sub>

closure (goto (I<sub>7</sub>, \*)) = I<sub>5</sub>

closure (goto (I<sub>8</sub>, +)) = I<sub>4</sub>

closure (goto (I<sub>8</sub>, \*)) = I<sub>5</sub>

state	Action						goto E
	id	+	*	(	)	\$	
0	S <sub>3</sub>			S <sub>2</sub>			1
1		S <sub>4</sub>	S <sub>5</sub>			accept	
2	S <sub>3</sub>			S <sub>2</sub>			6
3		r <sub>4</sub>	r <sub>4</sub>			r <sub>4</sub>	
4	S <sub>3</sub>			S <sub>2</sub>			7
5	S <sub>3</sub>			S <sub>2</sub>			8
6		r <sub>4</sub>	S <sub>5</sub>		S <sub>9</sub>		
7		S <sub>4</sub> or r <sub>1</sub>	S <sub>5</sub> or r <sub>1</sub>			r <sub>1</sub>	
8		S <sub>4</sub> or r <sub>2</sub>	S <sub>5</sub> or r <sub>2</sub>			r <sub>2</sub>	
9		r <sub>3</sub>	r <sub>3</sub>			r <sub>3</sub>	

- ①  $E \rightarrow E + E$        $FIRST(E) = \{ (, id \}$
  - ②  $E \rightarrow E * E$        $FOLLOW(E) = \{ \$, +, *, \epsilon \}$
  - ③  $E \rightarrow (E)$        $I_1, I_3, I_7, I_8, I_9$
  - ④  $E \rightarrow id$
- $I_1: E \rightarrow E + E.$   
 $I_8: E \rightarrow E * E.$   
 $I_9: E \rightarrow (E).$

Taking an input string

	vp	Action
\$0	id + id * id \$	S <sub>3</sub>
\$0\$id3	+ id * id \$	r <sub>4</sub> E → id
\$0E1	+ id * id \$	S <sub>4</sub>
\$0E1+4	id * id \$	S <sub>3</sub>
\$0E1+4id3	* id \$	Reduce r <sub>4</sub> E → id

\$OEI + 4E7

\*id \$

we choose  
Shift  $s_5$

\$OEI + 4E7 \* 5

id \$

Shift  $s_3$

\$OEI + 4E7 \* 5 id 3

\$

$r_4$   
 $E \rightarrow id$

\$OEI + 4E7 \* 5 E 8

\$

reduce  
 $r_1$   
 $E \rightarrow E * E$

\$OEI + 4E7

\$

$r_1$   
 $E \rightarrow E + E$   
accept

\$OEI

\$

For id \* id + id \$

Stack

input

Action



Using dangling - else Ambiguous Grammar -

$S \rightarrow iSeS / iS / a$

stmt  $\rightarrow$  if expression then stmt else stmt

stmt  $\rightarrow$  if expression then stmt

stmt  $\rightarrow$  for all other conditions

Construct SLR parsing table for the grammar  $\uparrow$

$S' \rightarrow \cdot S$

$S \rightarrow \cdot iSeS$

$S \rightarrow \cdot iS$

$S \rightarrow \cdot a$

$I_0 := \text{closure}(\text{goto}(I_0, S'))$

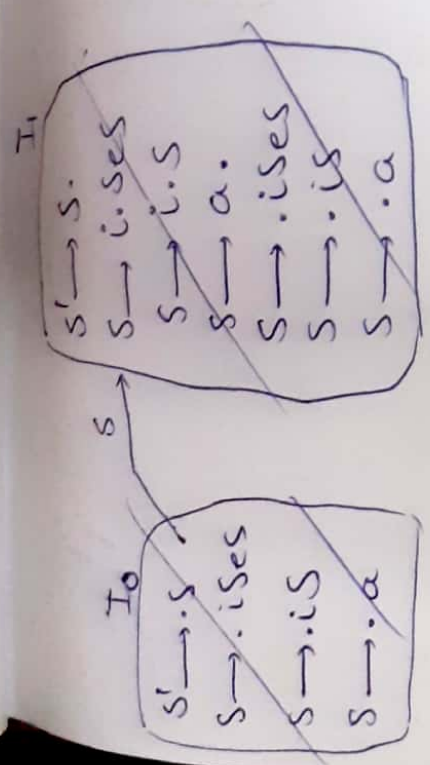
~~state~~     ~~action~~     ~~goto~~

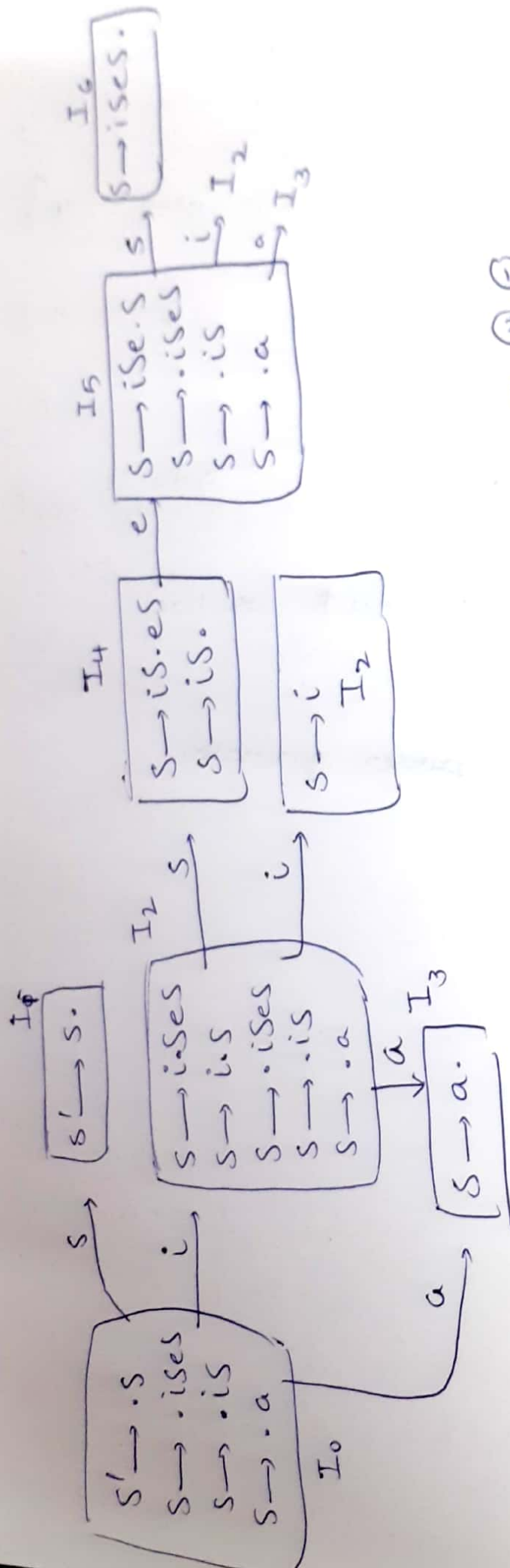
id + \* (

Parse the string  $iaea\$$  as the input string:

Note:

YACC is a ~~LALR~~ LALR parser generator





- ①  $S \rightarrow ises$
- ②  $S \rightarrow is$
- ③  $S \rightarrow a$

$I_1, I_3, I_4, I_6$

$I_1 :- S' \rightarrow s$

$I_3 :- S \rightarrow a$

Follow(S) =

$\{e, \$\}$

$I_4 :- S \rightarrow is$

$I_6 :-$

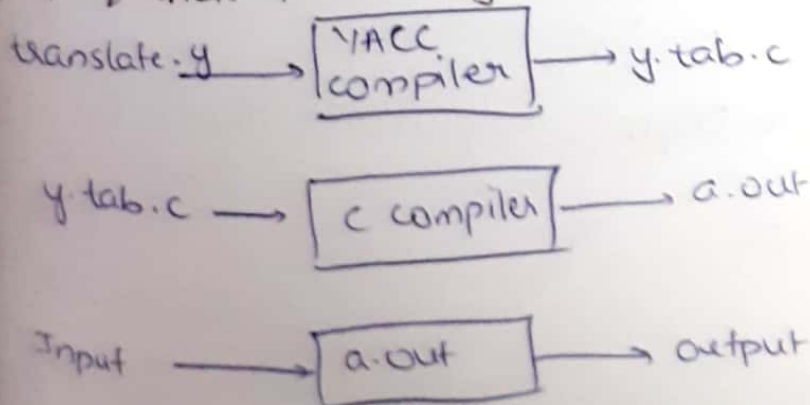
$S \rightarrow ises$

State	Action				goto
	i	a	e	\$	
0	S <sub>2</sub>	S <sub>3</sub>			S
1				accept	
2	S <sub>2</sub>	S <sub>3</sub>			4
3			r <sub>3</sub>	r <sub>3</sub>	
4			S <sub>5</sub> or r <sub>2</sub>	r <sub>2</sub>	
5	S <sub>2</sub>	S <sub>3</sub>			G
6			r <sub>1</sub>	r <sub>1</sub>	

Input symbols:-

\$0	iaea\$	shift S <sub>2</sub>
\$0i2	iaea\$	shift S <sub>2</sub>
\$0i2i2	aeaf\$	shift S <sub>3</sub>
\$0i2i2a3	ea\$	Reduce r <sub>3</sub> S → a
\$0i2i2S4	ea\$	shift S <sub>5</sub>
\$0i2i2\$4e5	a\$	shift S <sub>3</sub>
\$0i2i2S4e5a3	\$	Reduce r <sub>3</sub> S → a
\$0i2i2S4e5S6	\$	Reduce r <sub>1</sub> S → ises
\$0i2S4	\$	r <sub>2</sub> S → is
\$0S1	\$	accept

YACC :  $\Phi$  LALR parser generator.





Q. write a YACC compiler for desk calculator  
grammar for

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid \text{id}$

%.{

# include <ctype.h>

%.}

%. token DIGIT

%. .

line : expr '\n' { printf ("%d\n", \$1); }

;

expr : expr '+' term { \$\$ = \$1 + \$3 ; }

| term

;

term : term '\*' factor { \$\$ = \$1 \* \$3 ; }

| factor

;

factor : '(' expr ')' { \$\$ = \$2 ; }

| DIGIT

;

%. .

yylex()

{

int c;

c = getchar();

if (isdigit(c))

{ yylval = c - '0';

return DIGIT;

}

return c;

}

# Error recovery in LR PARSERS :-

- $E \rightarrow E + E$
- $E \rightarrow E * E$
- $E \rightarrow (E)$
- $E \rightarrow id$

	id	+	*	(	)	\$	E
0	$S_3$	$e_1$	$e_1$	$S_2$	$e_2$	$e_1$	1
1	$e_3$	$S_4$	$S_5$	$e_3$	$e_2$	accept	
2	$S_3$	$e_1$	$e_1$	$S_2$	$e_2$	$e_1$	6
3	$r_4$	$r_4$	$r_4$	$r_4$	$r_4$	$r_4$	
4	$S_3$	$e_1$	$e_1$	$S_2$	$e_2$	$e_1$	7
5	$S_3$	$e_1$	$e_1$	$S_2$	$e_2$	$e_1$	8
6	$e_3$	$S_4$	$S_5$	$e_3$	$S_9$	$e_4$	
7	$r_1$	$r_1$	$S_5$	$r_1$	$r_1$	$r_1$	
8	$r_2$	$r_2$	$r_2$	$r_2$	$r_2$	$r_2$	
9	$r_3$	$r_3$	$r_3$	$r_3$	$r_3$	$r_3$	

Step 1: Fill all the empty entries with the reduction value.

$$E \rightarrow \cdot E$$

Stack	Input	Error function
\$	+	Missing operand
\$	*	Missing operand
\$	\$	Missing operand
\$	)	Unbalanced Right Parenthesis

Step 2: Fill error messages in respective operators

Stack	Input	Error Action
\$...id	id	Missing operator
\$...id	(	Missing operator

step 3:

$$E \rightarrow (E \cdot)$$

↓  
id

stack	i/p
\$... (id	id
\$... (id	\$
\$... (id (	id

ex: "missing right parenthesis"

error Action

"missing operator"

"missing right parenthesis"

"missing operator"



## Unit III

### Syntax Directed Translation :-

Attributes are assigned as a property for each grammar symbol in the production. These attributes are of 2 types :-

#### Synthesized attributes :-

If the attributes in the grammar symbols on the left side of the production are computed using the attributes in the grammar. Then, that attributes are called as synthesized attributes.

If the attributes of the grammar symbol depends on the right side of the production are computed ~~as~~ using the attributes of the grammar symbols on the left side, then, the attributes are called as inherited attributes.

Attribute grammar :- Its a syntax directed definition in which the function in semantic rule is written as expressions attached with attributes.

Production

$$E \rightarrow E + T$$

$$E \rightarrow T$$

Semantic Rules

$$E.val \rightarrow E_1.val + T.val$$

$$E.val \rightarrow T.val$$

$$T \rightarrow T * F$$

$$T \rightarrow F$$

$$F \rightarrow (E)$$

$$F \rightarrow \text{digit}$$

$$T.\text{val} \rightarrow T_1.\text{val} * F.\text{val}$$

$$T.\text{val} \rightarrow F.\text{val}$$

$$F.\text{val} \rightarrow E.\text{val}$$

$$F.\text{val} \rightarrow \text{digit}.\text{lexval}$$

Translating the syntactic construct in the source code using their attributes is called as directed

SDD : Syntax directed definition:-

In SDD, each grammar production  $A \rightarrow \alpha$  is associated with semantic, a semantic rules of the form  $b = f(c_1, c_2, \dots, c_n)$  where  $f$  is a function &  $b$  is either inherited attribute or synthesized attribute.

(i) S-attributed definition:- If SDD uses only synthesized attribute then it is called as S-attributed definition.

If SDD uses, only inherited attributes, then it is called as L-attributed definitions.

Consider an example:-

$$E \rightarrow E + T$$

$$E \rightarrow T$$

$$T \rightarrow T * F$$

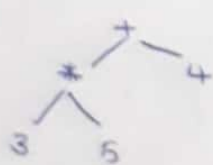
$$F \rightarrow F$$

$$F \rightarrow (E)$$

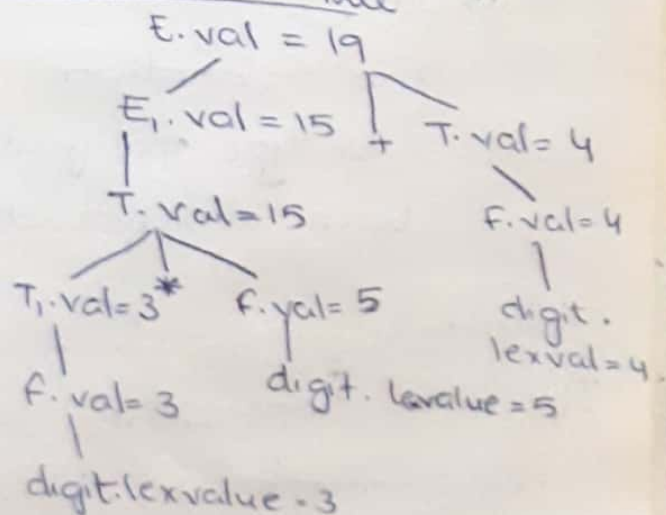
$$F \rightarrow \text{digit}$$

then construct syntax tree, parse tree & annotated parse tree for the string  $3 * 5 + 4$

Syntax tree:- based on precedence of operators



Annotated Parse Tree



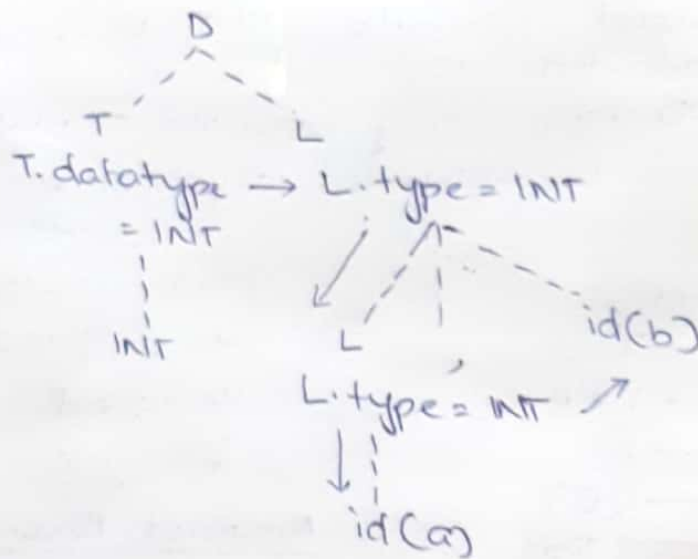
Annotated Parse Tree :-

Computing the parse tree, showing the values of the attributes at each node is called annotated parse tree or decorated parse tree.

Inherited attribute :

Production	Semantic Rules.
$D \rightarrow TL$	$L.type := T.datatype$
$T \rightarrow INT$	$T.datatype := INT$
$T \rightarrow FLOAT$	$T.datatype := FLOAT$
$T \rightarrow CHAR$	$T.datatype := CHAR$
$L \rightarrow L_1, id$	$L_1.type = L.type$ $insert(id-entry, L.type)$
$L \rightarrow id$	$insert(id-entry, L.type)$

Annotated parse tree for 'int a, b'



where,

D is declaration

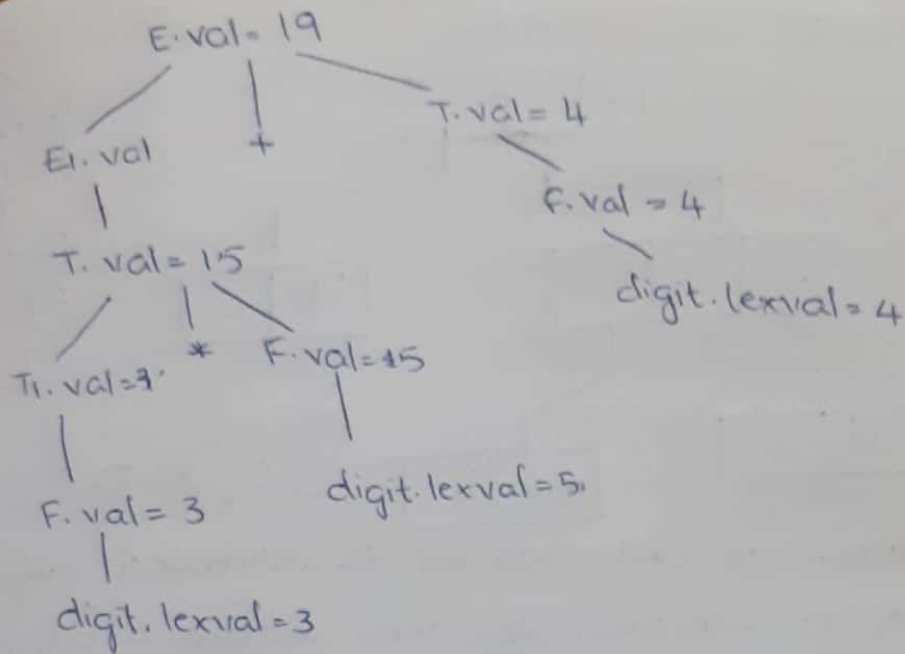
T is type specification

L is identifier list

Dependency graph :-

If b is dependent on a, it is represented by  $a \rightarrow b$





Construction of Syntax Tree :-

3 functions :-

1) mknode (op, left, right)

2) mkleaf (id, entry)

3) mkleaf (num, value)

Production

$E \rightarrow E_1 + T$

$E \rightarrow E_1 - T$

$E \rightarrow T$

$T \rightarrow (E)$

$T \rightarrow id$

$T \rightarrow num$

Semantic rule

$E.nptr = \text{mknode}('+', E_1.nptr, T.nptr)$

$E.nptr = \text{mknode}('-', E_1.nptr, T.nptr)$

$E.nptr = T.nptr$

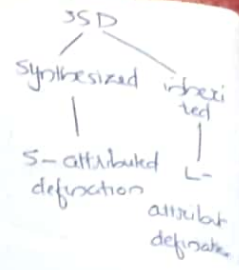
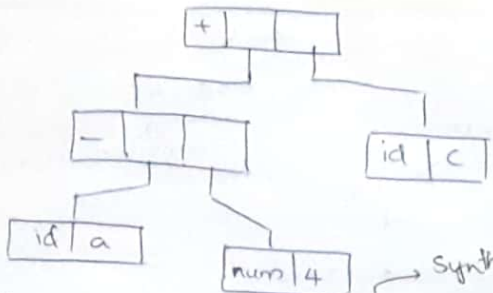
$T.nptr = E.nptr$

$T.nptr = \text{mkleaf}('id', id.entry)$

$T.nptr = \text{mkleaf}('num', value)$

Example :-

a-4+c



Bottom-up evaluation of S-attributed definition:-  
 Consider an example that reveals a grammar  
 which represents the implementation of  
 desk calculator with an LR Parser

Production	Semantic Rule
$L \rightarrow E_n$	print(E.val)
$E \rightarrow E_1 + T$	$E.val \rightarrow E_1.val + T.val$
$E \rightarrow T$	$E.val \rightarrow T.val$
$T \rightarrow T_1 * F$	$T.val \rightarrow T_1.val * F.val$
$T \rightarrow F$	$T.val \rightarrow F.val$
$F \rightarrow (E)$	$F.val \rightarrow E.val$
$F \rightarrow digit$	$F.val \rightarrow digit \text{ level}$

$T * F$	3 op 5	$T \rightarrow T * F$
$T$	3 * 5	$E \rightarrow T$
$E +$	15 _	
$E + 4$	15_4	$F \rightarrow digit$
$E + F$	15_4	$T \rightarrow F$
$E + T$	15_4	$E \rightarrow E + T$
$E$	15 + 4	
$E$	19	
$E_n$	19	
$L$	19	$L \rightarrow E_n$

```

code fragment
print(val[top])
val[top] = val[top-2] + val[top]

val[top] = val[top-2] * val[top]

val[top] = val[top-1]
    
```

Using above code fragment translate a stmt  
 3+5+4n and show the translation scheme in  
 detail

Input	State	val	Production rule
3*5+4n	-	-	
*5+4n	3	3	$F \rightarrow digit$
*5+4n	F	3	$T \rightarrow F$
*5+4n	T	3	
5+4n	T*	3 op	
+4n	T*5	3 op 5	$F \rightarrow digit$

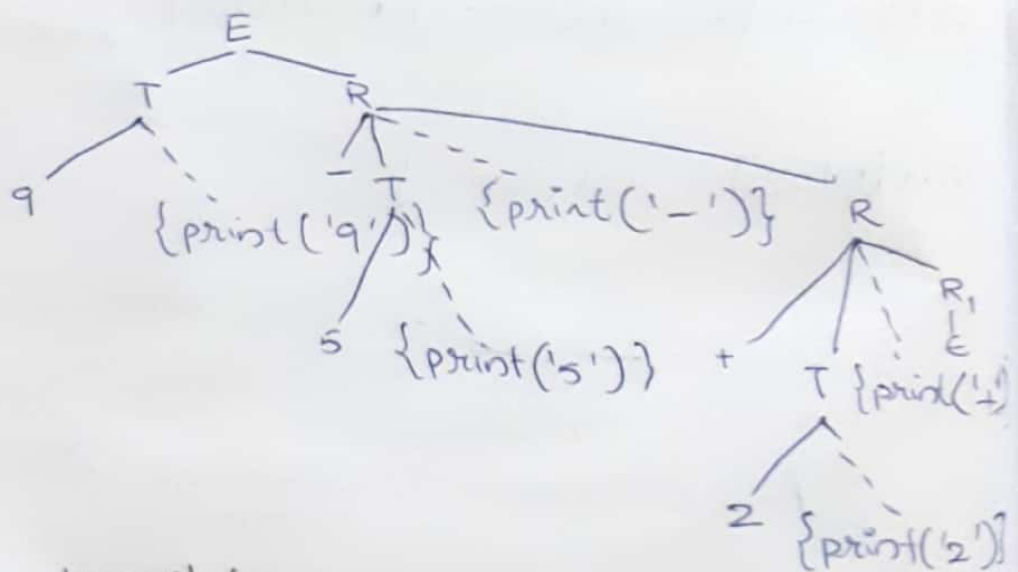
Top down evaluation of attributed definition (OR)  
 Translation scheme for inherited attribute grammar  
 Consider an example which includes a translation  
 scheme that maps infix expression with  
 addition & subtraction into corresponding  
 postfix expression.

Grammar

- ①  $E \rightarrow TR$
- $R \rightarrow \text{addop } T \{ \text{print}(\text{addop.lexeme}) \} R_1 / \epsilon$
- $T \rightarrow \text{num} \{ \text{print}(\text{num.val}) \}$

Construct the parse tree for the i/p  
 string for the i/p string  $9-5+2$  with each  
 semantic action attach.

~~$E \rightarrow TR$~~   
 ~~$R \rightarrow \text{addop } T \{ \text{print}(\text{addop.lexeme}) \} R_1 / \epsilon$~~   
 ~~$T \rightarrow \text{num}$~~



Top down translations :-

$$E \rightarrow E_1 + T \quad \{ E.val := E_1.val + T.val \}$$

$$E \rightarrow E_1 - T \quad \{ E.val := E_1.val - T.val \}$$

$$E \rightarrow T \quad E.val \rightarrow T.val$$

$$T \rightarrow (E) \quad T.val \rightarrow E.val$$

$$T \rightarrow \text{num} \quad T.val \rightarrow \text{num.lexval}$$



$E \rightarrow T \{ R.l := T.val \}$   
 $R \{ E.val := R.s \}$

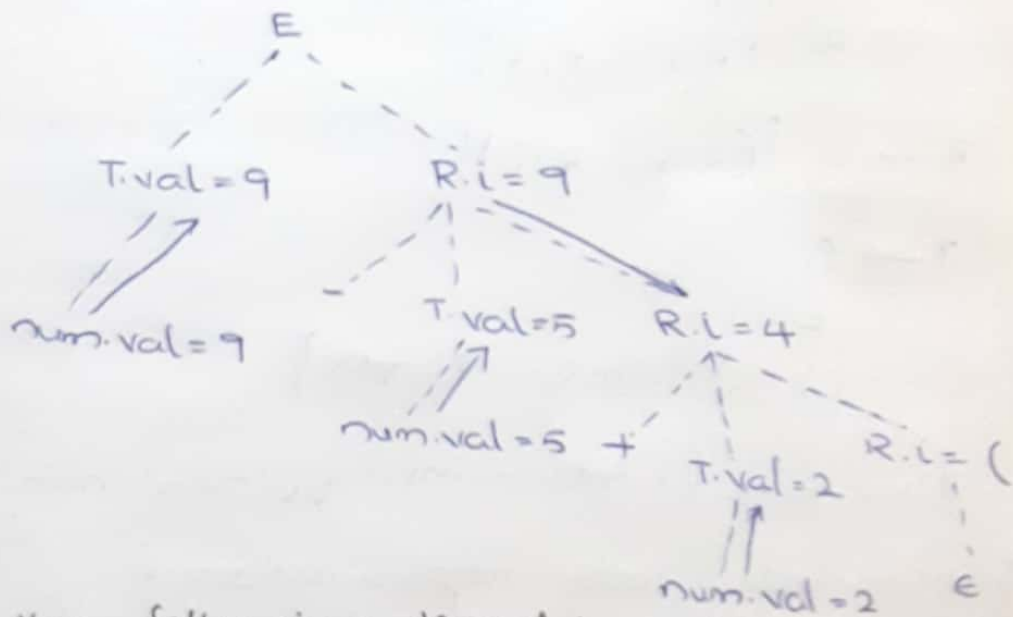
$R \rightarrow +$   
 $T \{ R.l := R.l + T.val \}$   
 $R.l \{ R.s := R.l \}$

$R \rightarrow -$   
 $T \{ R.l := R.l - T.val \}$   
 $R.l \{ R.s := R.l \}$

$R \rightarrow E \{ R.s := R.l \}$

$T \rightarrow ($   
 $E$   
 $) \{ T.val := E.val \}$

$T \rightarrow num \{ T.val := num.val \}$



consider the following translation scheme to construct the syntax tree.

$E \rightarrow E_1 + T / E_1$

$E \rightarrow TR / ER$

$R \rightarrow +TR / -TR / E$

Consider the following

$$E \rightarrow E_1 + T \quad \{ E.\text{nptra} = \text{mknode}(+, E_1.\text{nptra} + T.\text{nptra}) \}$$

$$E \rightarrow E_1 - T \quad \{ E.\text{nptra} = \text{mknode}(-, E_1.\text{nptra} + T.\text{nptra}) \}$$

$$E \rightarrow T \quad \{ E.\text{nptra} = T.\text{nptra} \}$$

$$T \rightarrow (E) \quad \{ T.\text{nptra} = E.\text{nptra} \}$$

$$T \rightarrow \text{num} \quad \{ T.\text{nptra} = \text{mkleaf}(\text{num}, \text{num.val}) \}$$

$$E \rightarrow T \quad \{ R.i := T.\text{nptra} \}$$

$$R \quad \{ E.\text{nptra} := R.s \}$$

$$R \rightarrow +$$

$$T \quad \{ R_1.i := \text{mknode}('+', R.i, T.\text{nptra}) \}$$

$$R_1 \quad \{ R.s := R_1.s \}$$

$$R \rightarrow -$$

$$T \quad \{ R_1.i := \text{mknode}('-', R.i, T.\text{nptra}) \}$$

$$R_1 \quad \{ R.s := R_1.s \}$$

$$R \rightarrow E \quad \{ (R.s := R.i) \}$$

$$T \rightarrow ($$

E

$$) \quad \{ T.\text{nptra} := E.\text{nptra} \}$$

$$T \rightarrow \text{num} \quad \{ T.\text{nptra} := \text{mkleaf}('num', \text{num.val}) \}$$

$T \rightarrow \text{num} \{ T.\text{opte} := \text{mkleaf}('num', \text{num.val}) \}$

Intermediate code representation :-

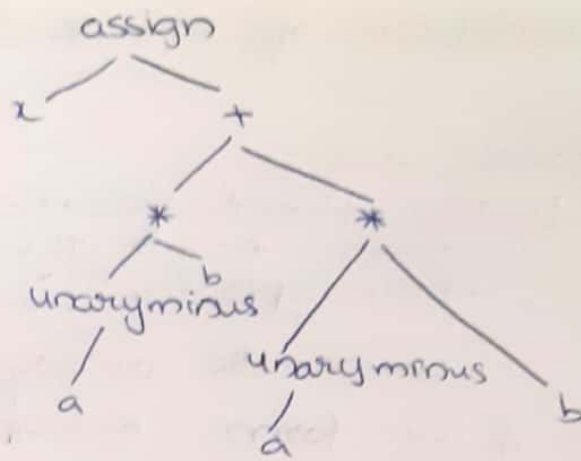
Intermediate code can be represented in 3 ways :-

- (i) Abstract syntax tree
- (ii) postfix expression
- (iii) 3-address code

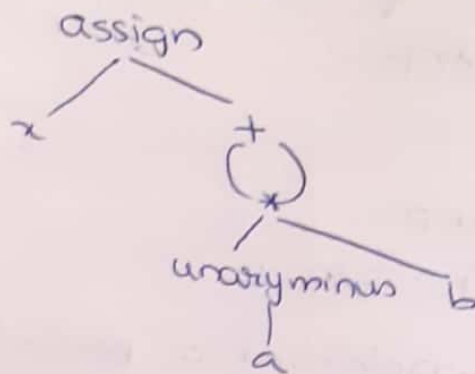
Consider an ifp string  $x := -a*b + -a*b$



1) Abstract syntax tree



There is repetition of  $-a * b$  to avoid this we use DAG (Directed acyclic graph)



(2) Postfix expression :-

$$(a+b) * c = ab+c *$$

ex:  $(3+4) * 5$

$$34 + 5 *$$

(3) 3-address code :-

The reason why it is called 3-address code is, the statement usually contains 3 addresses, where 2 addresses are used to store the operands and one address to store the result

$$x = y * z$$

$$\begin{cases} t_1 = y * z \\ x = t_1 \end{cases}$$

3 addresses

1. one to store results
2. the rest to store operands

## General representation of 3-address code :-

$A := B \text{ op } C$

Types of 3-address code :-

1)  $A := B \text{ op } C$

2)  $A := \text{op } B$

~~3)  $A := B$~~

(1) Assignment statements of the form  $A := B \text{ op } C$

where operator can be binary arithmetic or logical operator

(2) Assignment is of the form  $A := \text{op } B$  where op is of a unary operator, logical negation or shift

(3) Copy statements of the form  $A := B$

(4) Unconditional jump  
goto L

(5) Conditional jump  
if A relop B goto L

(6) Instruction to implement a procedure call.  
param A and call P, n, where n is number of actual parameters

param A<sub>1</sub>  
param A<sub>2</sub>

param A<sub>n</sub>

call P, n

return B

(7) Indexed assignments

$A := B[i]$

(or)  $A[i] := B$

(8) Address & pointer assignments

$A := \&B$

$A := *B$

$*A := B$

ex: - The 3 address code for  $A = -b * (c + d)$

$t_1 := \text{uminus } b$

$t_2 = c + d$

$t_3 = t_1 * t_2$

$A := t_3$

ex2:  $a := -b * c + -b * c$

$t_1 = \text{uminus } b$

$t_2 = t_1 * c$

$t_3 := -b$

$t_4 = t_3 * c$

$t_5 = t_2 + t_4$

$a := t_5$

Three forms of representing 3 address code are :-

- (i) Quadruples
- (ii) Triples
- (iii) Indirect triples

Quadruples :-

	op	arg1	arg2	result
(0)	uminus	b		$t_1$
(1)	*	$t_1$	c	$t_2$
(2)	uminus	b		$t_3$
(3)	*	$t_3$	c	$t_4$
(4)	+	$t_2$	$t_4$	$t_5$
(5)	:=	$t_5$		a

Triples :-

In triple representation of 3-address code we don't have result column, so the evaluation will be expressed where the results evaluation is expr are stored in a particular position or location in the memory



## 2) Triples

	op	arg1	arg2
(0)	uminus	b	
(1)	*	(0)	<del>b</del> c
(2)	uminus	b	
(3)	*	(2)	c
(4)	+	(1)	(3)
(5)	:=	a	(4)

$$A = B[i]$$

$$A[i] = B$$

	op	arg	arg
(0)	[ ] =	B	i
(1)	:=	A	(0)

	op	arg1	arg2
(0)	[ ] =	A	<del>b</del> i
(1)	=	(0)	B

## (3) Indirect Triples :-

	Statement
(0)	→ (14)
(1)	→ (15)
(2)	→ (16)
(3)	→ (17)
(4)	→ (18)
(5)	→ (19)

	Triples		
	op	arg1	arg2
(14)	uminus	b	
(15)	*	<del>(14)</del>	c
(16)	uminus	b	
(17)	*	(16)	c
(18)	+	(15)	(17)
(19)	:=	a	(18)

• It is another form of representation of 3-address code where it lists out pointers to triples such that an array statement is used in order to list out pointers to triples in the desired order.

Example: write the 3 address code forms for quadruples, triples, indirect triples for

$$A := -B * (C + D)$$

$t_1 := -\text{uminus } b$

$t_2 := c + d$

$t_3 := t_1 * t_2$

$A := t_3$

(1) Quadruples :

	op	arg1	arg2	result
(0)	uminus	b		$t_1$
(1)	+	c	d	$t_2$
(2)	*	$t_1$	$t_2$	$t_3$
(3)	:=	$t_3$		A

(2) Triples :-

	op	arg1	arg2
(0)	uminus	b	
(1)	+	c	d
(2)	*	(0)	(1)
(3)	:=	A	(2)

(3) Indirect Triples :-

	Statement
(0)	→ (14)
(1)	→ (15)
(2)	→ (16)
(3)	→ (17)

	op	arg1	arg2
(14)	uminus	b	
(15)	+	c	d
(16)	*	(14)	(15)
(17)	:=	A	(16)

conversion of popular programming language constructs in 3-Address code

(1) Assignment statements

$$v_1 = (v_2 - v_3) * (v_2 + 2 * v_3)$$

$$t_1 := v_2 - v_3$$

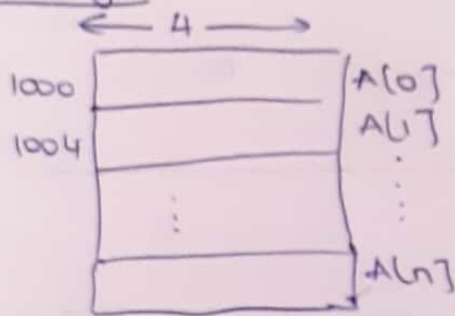
$$t_2 := 2 * v_3$$

$$t_3 := v_2 + t_2$$

$$t_4 := t_1 * t_3$$

$$v_1 := t_4$$

(2) Arrays :-



To find the location of

$A[i]$  the formula is

$$\text{base} + i * w$$

( $w = \text{width}$ )

$$1008 = 1000 + 2 * 4$$

Three Address Code

$$t_0 := 5 * 4$$

$$t_1 := \&a$$

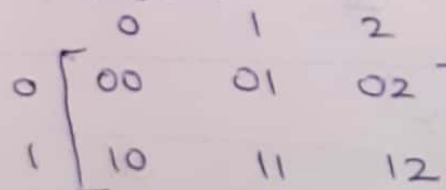
$$t_2 := t_1[t_0]$$

$$P := t_2$$

eg 1) `int a[10];`

stmt: `P = a[5];`

eg 2) `int a[2][3]`



Row Major form

1st row `a[0][0]`

`a[0][1]`

`a[0][2]`

`a[1][0]`

2nd row `a[1][1]`

`a[1][2]`

Column Major form

1st col `a[0][0]`

`a[1][0]`

2nd col `a[0][1]`

`a[1][1]`

3rd col `a[0][2]`

`a[1][2]`

datatype  $a[d_1][d_2]$   
 $a[i_1][i_2]$

Row major form  $\left\{ \begin{array}{l} \text{base} + (i_1 * d_2 * w + i_2 * w) \\ \text{or} \\ \text{base} + (i_1 * d_2 + i_2) * w \end{array} \right.$

column major form  $\left\{ \begin{array}{l} \text{base} + (i_1 * w + i_2 * w * d_1) \\ \text{or} \\ \text{base} + (i_1 + i_2 * d_1) * w \end{array} \right.$

eg:-  $\text{int } a[20][30]$   
 $P = a[5][9]$   
 $i_1 \quad i_2$

Row:-  $\text{base} + (5 * 30 + 9) * 4$   
 $\text{base} + (159) * 4$   
 $\text{base} + 636$

$\text{int } a[20][30]$   
Code

$P = a[5][9]$

Intermediate code form

$t_0 := 30 * 4$

$t_1 := t_0 * 5$

$t_2 := \&a$

$t_3 := 9 * 4$

$t_4 := t_1 + t_3$

$t_5 := t_2[t_4]$

$P := t_5$

eg:-  $\text{int } a[20][30]$   
 $a[4][3] = h$

Row:-  $\text{base} + (4 * 30 + 3) * 4$

Intermediate code form

$t_0 := 30 * 4$

$t_1 := t_0 * 4$

$t_2 := \&a$

$t_3 := 3 * 4$

$t_4 := t_1 + t_3$



## Pointers & Address assignments :-

①  $P = \&x;$   
 $*P = 40;$

$t_0 := \&x;$   
 $P := t_0$   
 $P[0] := 40;$

} Intermediate code.

②  $P = \&x;$   
 $y = *P;$

$t_0 := \&x$   
 $P := t_0$   
 $t_1 := P[0];$   
 $y := t_1$

③  $P = \&a[5];$   
 $*P = 50;$

$t_0 := 5 * 4$   
 $t_1 := \&a$   
 $t_2 := t_0 + t_1$   
 $P := t_2$   
 $P[0] := 50$

## Record Access :-

structures :-

struct Students

{  
  int stud\_id;  
  int age;  
}\*s<sub>1</sub>;  
  ⋮

$s_1.age := 18;$

$s_1 \rightarrow age := 18;$

↓  
 $t_0 := \&s_1;$   
 $t_1 := 4;$   
 $t_0[t_1] := 18$

Intermediate code

$t_0 := \&s_1;$   
 $t_1 := 4;$   
 $t_0[t_1] := 18;$

(or)

$t_0 := \&s_1;$   
 $t_0[4] := 18;$

} Intermediate code form

Flow of control statements & boolean expressions

1) If-else statements

```

1) if (a > b)
    c = 40;
else
    c := 30;

x = 50;
    
```

```

if (a > b) goto label L0
goto L1
Label L0
c := 40
goto label L2
Label L1
c := 30
goto label L2
Label L2
x := 50;
    
```

2) if (a > b) || (b > c)

```

a = 40;
else
    a := 20;
    
```

Intermediate code form

```

if (a > b) || (b > c) goto L0
goto L1
Label L0
a := 40
Label L1
a := 20
    
```

3) switch case statement :-

```

switch (ch)
{
case 1: a = b;
        c = d;
        break;
case 2: e := f;
        g := h;
        break;
default: a = b;
         c = d;
         break;
}
p = q;
    
```

```

to := ch
goto label L3
Label L0
a := b
goto L4
Label L1
e := f
g := h
goto L4
Label L2
a := b
c := d
goto L4
Label L3
if to == 1 goto L0
if to == 2 goto L1
goto L2
Label L4
p := q
    
```

Loop statements :-

while (i > 0)

```
{  
  val := val * i;  
  i := i - 1;  
}
```

=

Label L<sub>0</sub>

```
if (i > 0) goto L1  
goto L2
```

Label L<sub>1</sub>

```
to := val * i  
val := to  
t1 := i - 1  
i := t1
```

goto L<sub>0</sub>

Label L<sub>2</sub>

=

Procedure call (parameters)

1) p = sum(n<sub>1</sub>, n<sub>2</sub>)

Intermediate code form :

calling sequence & return

param n<sub>1</sub>

param n<sub>2</sub>

call sum s

↓  
where s is size of n<sub>1</sub> +  
size of n<sub>2</sub>

For returning

return to;

p := to

ex:-  
2) sum(int n<sub>1</sub>, int n<sub>2</sub>)

```
{  
  int s;  
  s = n1 + n2;  
  return(s);  
}
```

Intermediate code:

TAC: proc-begin sum

to := n<sub>1</sub> + n<sub>2</sub>

s := to

return s

goto L<sub>0</sub>

Label L<sub>0</sub>

proc-end sum

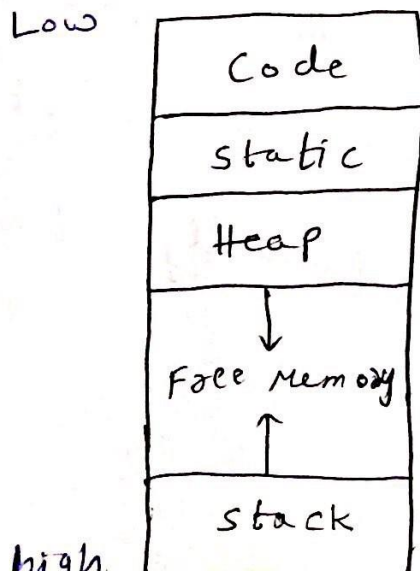
## Unit - IV

### Run time Environments

#### Storage Organization

- The management and organization of this logical address space is shared between the compiler, operating system and target machine.
- The operating system maps the logical addresses into physical addresses, which are usually spread throughout memory.
- The runtime representation of an object program in the logical address space consists of data and program areas.

Typical Subdivision of run-time Memory into code and data areas





→ Runtime storage comes in blocks of contiguous bytes, where a byte is the smallest unit of addressable memory.

→ A byte is eight bits and four bytes form a machine word.

→ Multiple byte objects are stored in consecutive bytes and given the address of the first byte.

→ The storage layout for data objects is strongly influenced by the addressing constraints of the target machine.

→ space left unused due to alignment considerations is referred to as padding.

→ The size of the generated target code is fixed at compile time, so the compiler can place the executable target code in a statically determined area Code, (usually low end of memory)

→ Similarly, the size of some program data objects, such as global constants, and data generated by the compiler, such as information to support garbage collection, may be known at compile time and these data objects can be placed in another statically determined area called

static.

→ One of the reasons for statically allocating as many data objects as possible is that the addresses of these objects can be compiled into the target code.

→ To maximize the utilization of space at run time, the other two areas, stack and heap are at the opposite ends of remainder of address space.

→ These <sup>areas</sup> are dynamic, their size can change as the program executes.

→ These areas grow towards each other as needed.



→ The stack is used to store data structures called activation records that get generated during procedure calls.

→ stack grows towards lower addresses, heap towards higher.

### Storage Allocation (Static vs Dynamic)

→ The layout and Allocation of data to memory locations in the runtime environment are key issues in storage management.

→ Storage allocation decision is static, if it can be made by compiler looking only at text of program, not at what program does when it executes.

→ Storage allocation decision is dynamic, if it can be decided only while the program is running.

→ stack storage - Names local to a procedure are allocated space on stack. The stack supports the normal call/return policy for procedures.

→ Heap storage : Data that may ~~not~~ outlive the call to the procedure that created it is usually allocated on a "heap" of reusable storage. The heap is area of virtual memory that allows ~~of~~ objects (or) other data elements to obtain storage when they are created and to return that storage when they are invalidated.

### Storage Allocation ~~strategies~~ strategies

→ There are three different type of storage allocation strategies based on the division of runtime storage. These are

- (i) static Allocation - It is ~~for~~ <sup>for</sup> all data objects at compile time
- (ii) stack Allocation - In the stack allocation a stack is used to manage runtime storage
- (iii) Heap Allocation - In heap allocation the heap is used to manage dynamic memory allocation.

### (i) Static Allocation

→ The size of data objects is known at compile time. The names of these objects



are bound to storage at compile time only and such an allocation of data objects is done by static allocation.

→ The binding of name with the amount of storage allocated don't change at run time.

→ In static allocation the compiler can determine the amount of storage required by each data object. Therefore it becomes easy for a compiler to find the addresses of these data in the activation record.

→ At compile time compiler can fill the addresses at which the target code can find the data it operates on.

→ FORTAN uses static allocation strategy.

Limitations of static Allocation

→ The static allocation can be done only if the size of data object is known at compile time.

→ The data structures can not be created dynamically. The static allocation can not manage the allocation of memory at runtime.

→ Recursive procedures are not supported by this type of allocation.

## (ii) stack Allocation

→ Stack Allocation strategy is a strategy in which the storage is organized as stack.

This stack is also called control stack.

→ As activation begins, the activation records are pushed onto the stack and on completion of this activation record the corresponding activation records are popped.

→ The locals are stored in each activation record. Hence locals are bound to corresponding activation record on each fresh activation.

→ The data structures can be created dynamically for stack allocation.



## Limitations of stack allocation

→ The memory addressing can be done using pointers and index registers. Hence this type of allocation is slower than static allocation.

## Heap Allocation

→ If the values of non-local variables must be retained even after the activation record then such a retaining is not possible by stack allocation. This limitation of stack allocation is because of its last In First out nature. For retaining of such local variables Heap Allocation strategy is used.

→ The Heap allocation allocates the continuous block of memory when required for storage of activation records (or) other data object.

→ This allocated memory can be deallocated when activation ends.

→ The deallocated (free) space can be further reused by heap manager.



→ The efficient heap Management can be done by

- i) creating a linked list for the free blocks and when any memory is deallocated that block of memory is appended in the linked list.
- ii) Allocate the most suitable block of memory from the linked list (i.e) use best fit technique for allocation of block.

### Stack Allocation of space

→ ~~At~~ → Almost all compilers for Languages that use procedures, functions (or) methods as units of user-defined actions manage at least part of their runtime memory as stack.

→ Each time a procedure is called, space for its local variables is pushed onto a stack, and when procedure terminates, that space is popped off the stack.

### Activation Trees

→ stack allocation would not be feasible if procedure calls (or) activations of procedures, did not nest in time.

Eg: quick sort (recursive)

```
int a[11];
```

```
void readArray() { /* Read 9 integers into  
a[1] ... a[9] */
```

```
int i;  
...  
}
```

```
int partition (int m, int n)
```

```
{
```

\* Picks a separator value  $v$ , and partitions

$a[m..n]$  so that  $a[m..p-1]$  are less than

$v$ ,  $a[p]=v$ , and  $a[p+1..n]$  are equal to

(\*) greater than  $v$ . Returns  $p$ . \*/

```
...  
}
```

```
{
```

```
void quicksort (int m, int n)
```

```
{
```

```
int i;
```

```
if (n > m)
```

```
{
```

```
i = partition (m, n);
```

```
quicksort (m, i-1);
```

```
quicksort (i+1, n);
```

```
}
```

```
}
```

```

main()
{
readArray();
a[0] = -9999;
a[10] = 9999;
quicksort(1, 9);
}

```

→ Sequence of calls that might result from an execution of the program.

```

enter main()
    enter readArray()
    leave readArray()
    enter quicksort(1, 9)
        enter partition(1, 9)
        leave partition(1, 9)
        enter partition(1, 3)
        ----
        ----
        leave partition(1, 3)
        enter partition(5, 9)
        ----
        ----
        leave partition(5, 9)
    leave quicksort(1, 9)
leave main()

```



→ In this example, as it is true in general, procedure activations are nested in time.

→ If an activation of procedure  $q$ , then that ~~act~~ activation of  $q$  must end before the activation of  $p$  can end.

These are three common cases:

(i) The activation of  $q$  terminates normally -

Then the control resumes just after the point of  $p$  at which the call to  $q$  was made -

(ii) The activation of  $q$ , (or) some procedure  $q$  called, either directly (or) indirectly, aborts i.e. it becomes impossible for execution to continue.

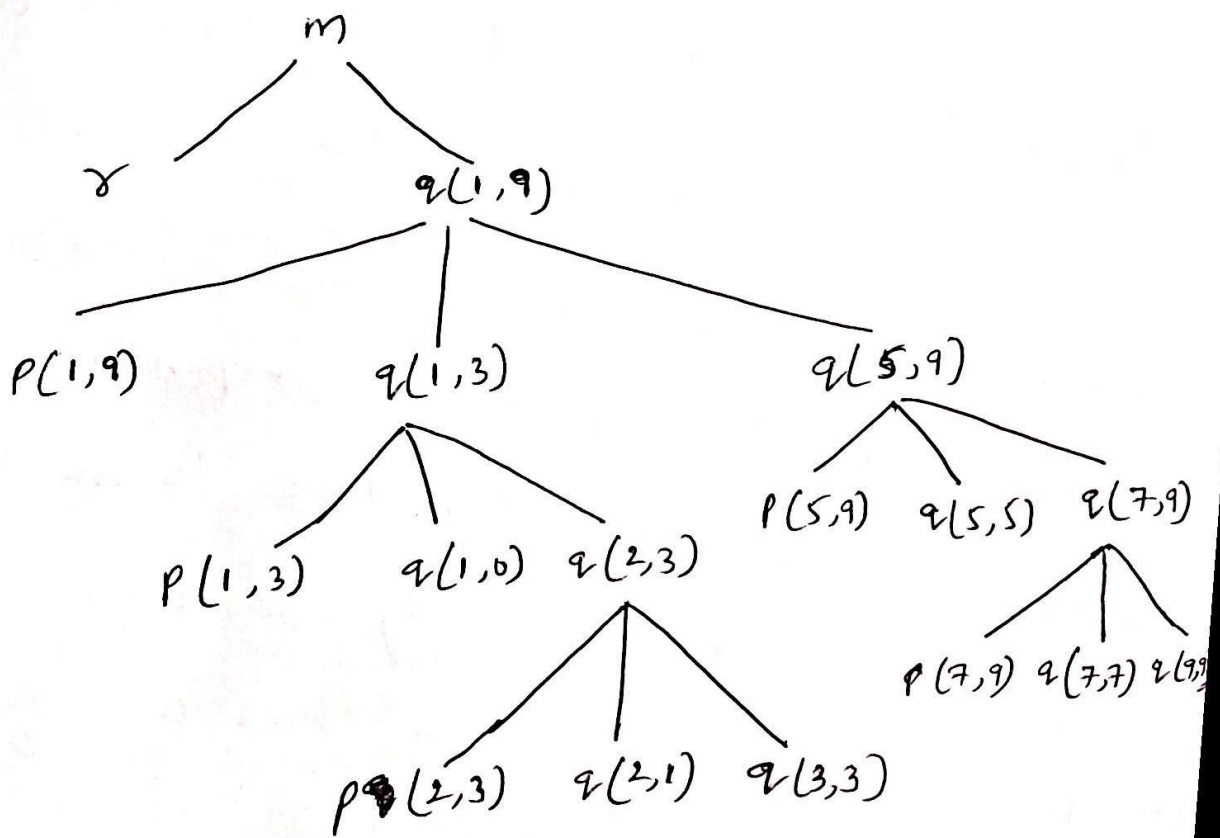
In that case,  $p$  ends simultaneously with  $q$ .

(iii) The activation of  $q$  terminates because of an exception that  $q$  cannot handle. Procedure  $p$  may handle the exception, in which case the activation of  $q$  has terminated while the activation of  $p$  continues.

→ we represent the activations of procedures during the running of entire program by a tree, called Activation Tree.

→ Each node corresponds to one activation, the root is the activation of the "main" procedure that initiates execution of program.

→ At a node for an activation of procedure P, the children correspond to activations of the procedures called by this activation of P.



Activation tree representing calls during an execution



- The sequence of procedure calls correspond to a preorder traversal of the activation tree.
- The sequence of returns corresponds to a postorder traversal of the activation tree.

### Activation Records

→ procedure calls and returns are usually managed by a runtime stack called the Control stack.

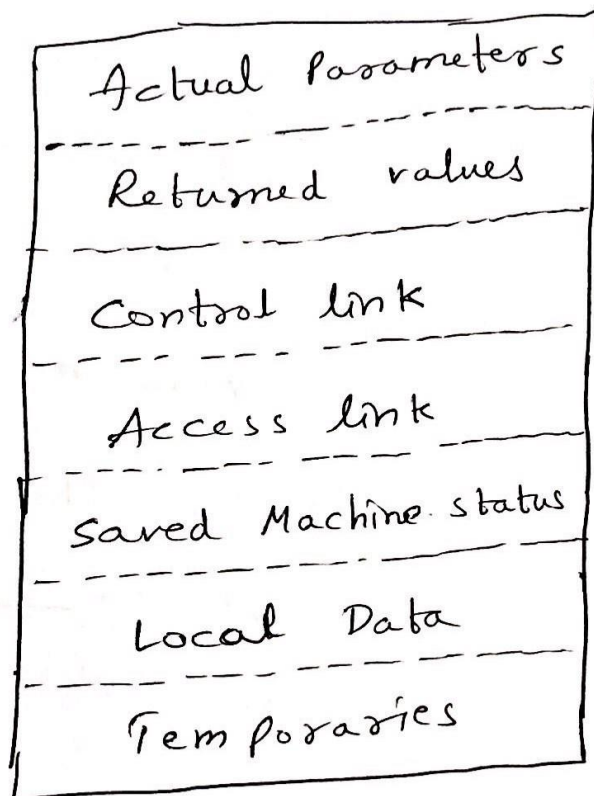
→ Each live activation has an activation record on the control stack, with the root of the activation tree at the bottom, and the entire sequence of activation records on the stack corresponding to the path in the activation tree to the activation where control currently resides.

→ The latter activation has its record at the top of the stack.



→ The Activation Record is a block of memory used for managing information needed by a single execution of a procedure.

→ The contents of activation records vary with the language being implemented.



A general Activation Record

→ Temporary values, such as those arising from the evaluation of expressions, in cases where those temporaries cannot be held in registers.

→ Local data belonging to the procedure whose activation record this is.

→ A saved Machine status, with information about the state of the machine just before the call to procedure. This information typically includes the return address and the contents of registers that were used by calling procedure and that must be restored when the return occurs.

→ An "access link" may be needed to locate data needed by the called procedure. This field refers to nonlocal data in another activation record. This field is also called static link field.

→ A "control link", pointing to the activation record of the caller. It points to activation record of calling procedure. This link is ~~also~~ called dynamic link.



→ space for return value of the called function

→ The actual parameters used by the calling procedure

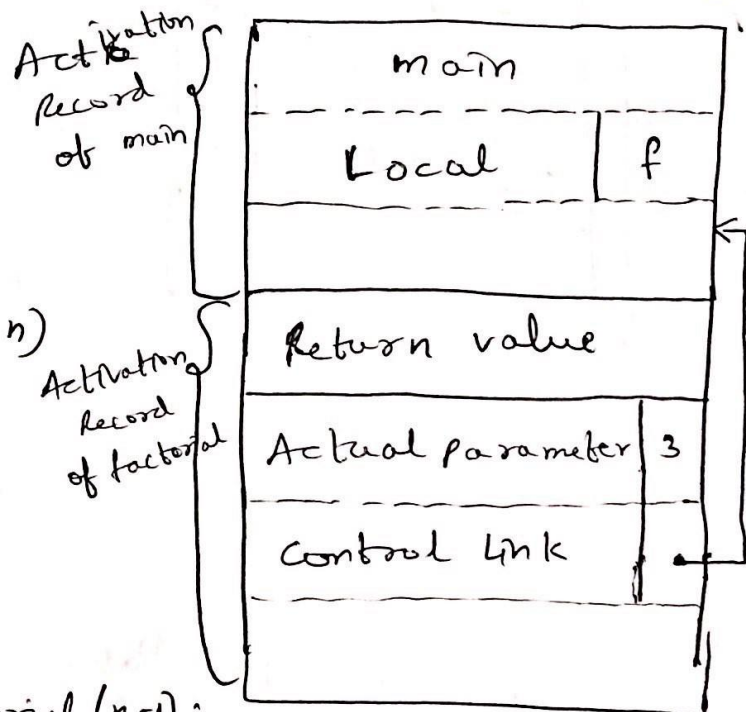
→ The size of each field of activation record is determined at the time when a procedure is called.

Eg:

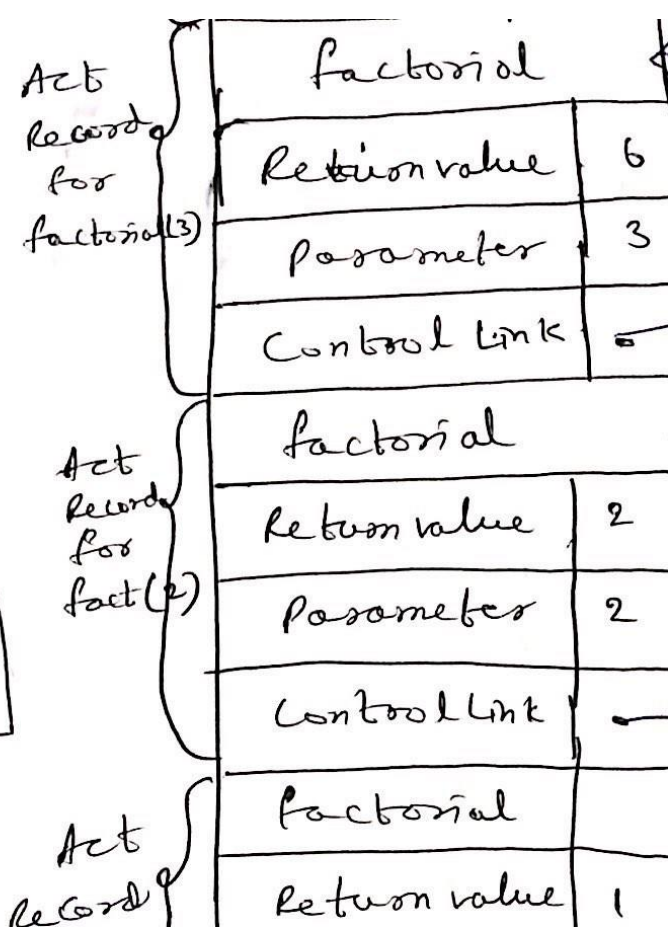
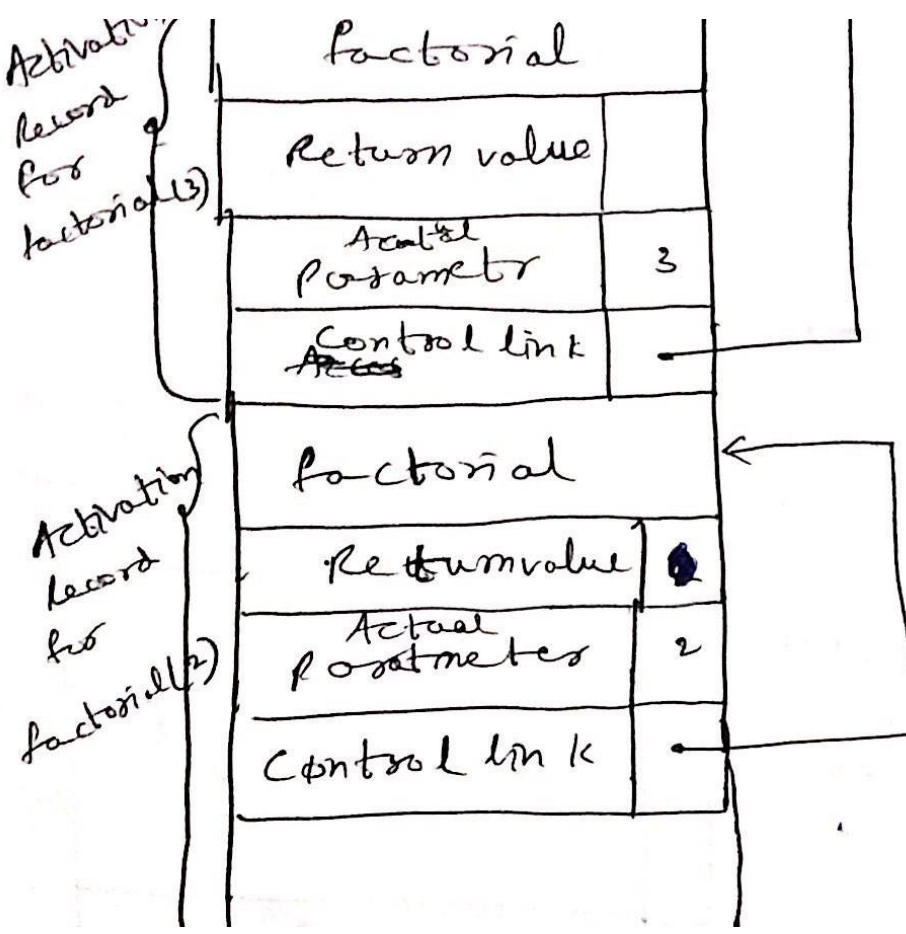
```
main()
{
  int f;
  f = factorial(3);
}

int factorial(int n)
{
  if (n == 1)
    return 1;
  else
    return n * factorial(n-1);
}
```

First call Activation Record







Eg:- Suppose a procedure A calls the procedure B (callee procedure). ~~begins after time array.~~

→ procedure A has two arrays x & y.

The storage of these arrays is not the part of activation ~~record~~ record of A.

→ In the activation record of A only the pointers to the beginning of x and y are appearing. We can obtain the relative addresses of these arrays at compile time.

→ The activation record of for procedure B begins after the arrays of A. Suppose the procedure

B has variable length arrays p and q.

Then after the activation record of B the array for procedure B can be placed.

→ Two pointers can be maintained top and top-sp to keep track of these records.

→ The top points to actual top of the stack and top-sp points to end of some field in the activation record.

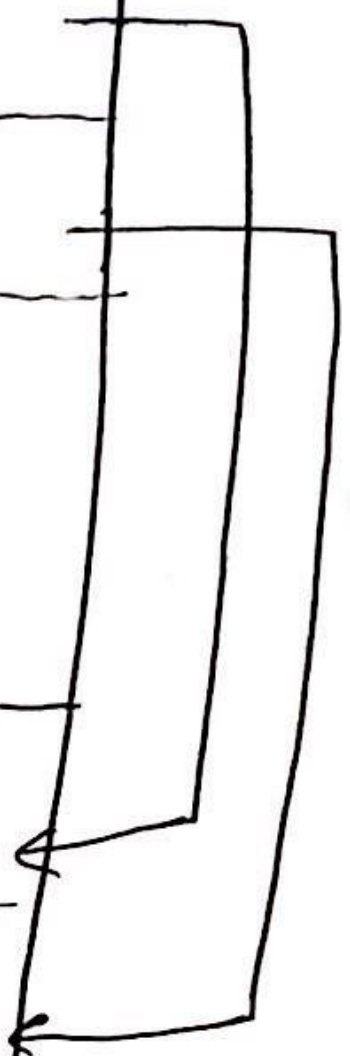
control link

pointer to x

pointer to y

array x

array y





## Access to ~~the~~ Non-Local Data on the stack

→ The storage Allocation can be done for two types of data variables.

(i) Local data

(ii) Non-Local data

→ The Local data can be handled using activation record whereas non local data can be handled using scope information

→ The block structured storage allocation can be done using static scope or lexical scope.

→ The non block structured storage allocation can be done using dynamic scope.

### Local data

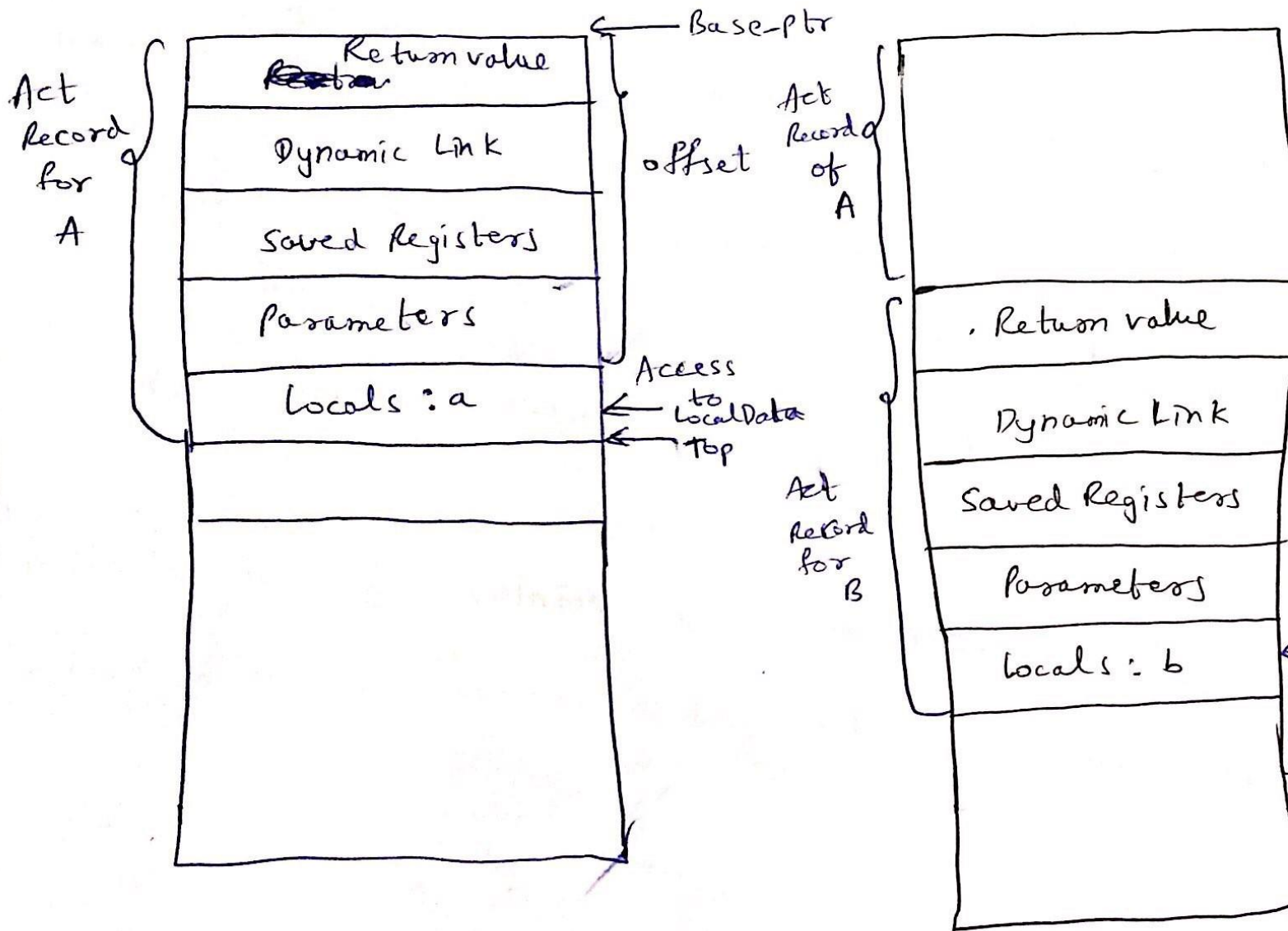
→ The local data can be accessed with the help of activation record.

→ The offset relative to base pointer of an activation record points to local data variables within activation record.

Eg:- Procedure A  
 int a;  
 procedure B  
 int b;  
 body of B;  
 body of A;

The contents of stack along with base pointer

offset



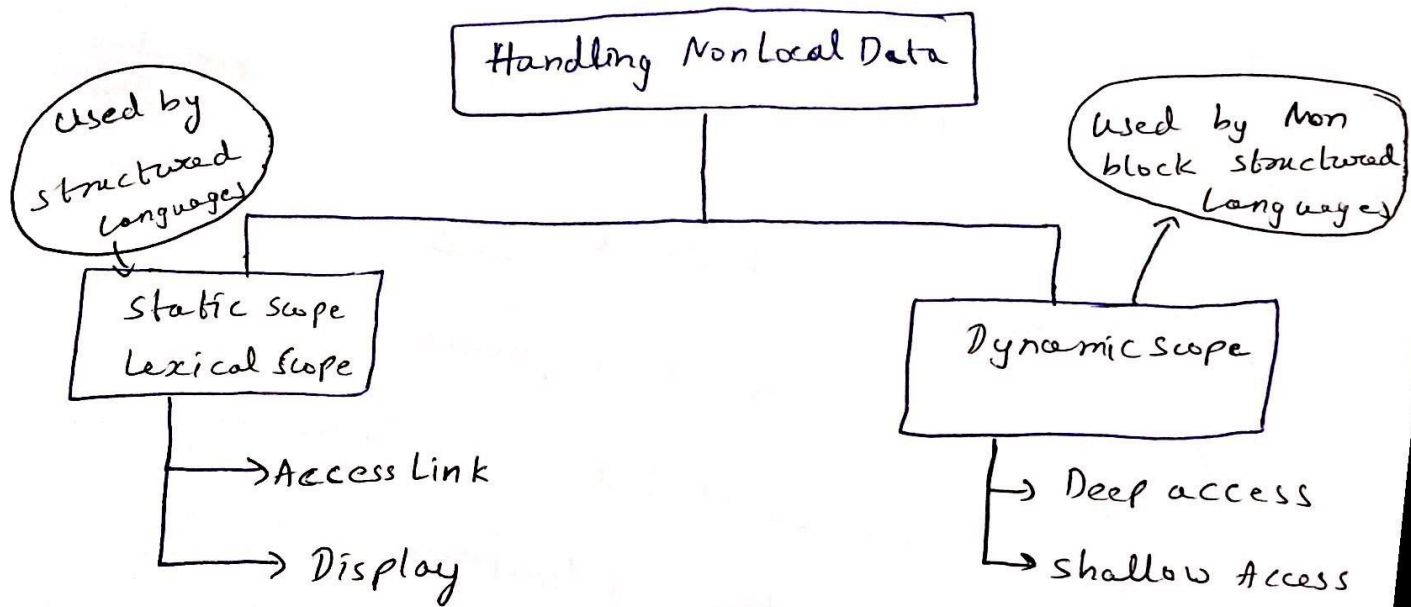
## Access to Non Local Names

→ A procedure may sometimes refer to variables which are not local for it. Such variables are called as Non Local variables.

→ For the Non-local Names there are two types of scope rules that can be defined

(i) static

(ii) dynamic



### Static Scope Rule

→ It is also called as lexical scope. In this type the scope is determined by examining the program text.

→ PASCAL, C and ADA use static scope rule.



→ These Languages are also called as Block Structured Languages.

(ii) Dynamic Scope rule

→ For non block structured Languages this dynamic scope allocation rules are used.

→ The dynamic scope rule determines the scope of declaration of names at runtime by considering the current activation.

Eg: LISP and SNOBOL use Dynamic Scope rule.

Static Scope (or) Lexical Scope

→ Block :- It is a sequence of statements containing the local data declarations and enclosed within the delimiters.

Eg: 
$$\left. \begin{array}{l} \text{Declaration statements;} \\ \dots \end{array} \right\}$$

→ The delimiters mark the beginning and end of the Block. The Blocks can be in nesting fashion that

means block  $B_2$  completely can be inside the block  $B_1$ .

→ The scope of declaration in a block structured language is given by most closely nested loop (or) static rule.

→ The declarations are visible at a program point are:

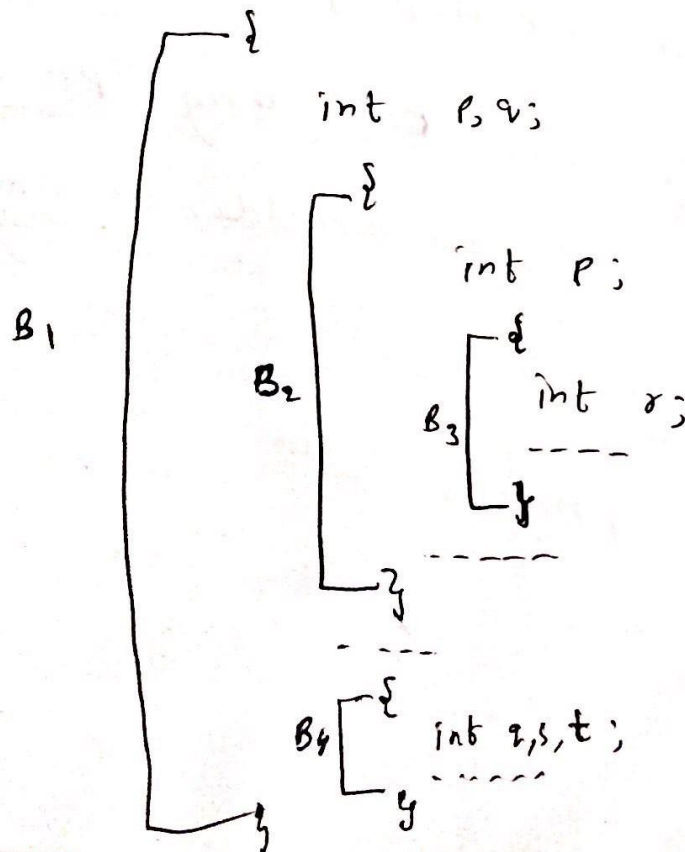
(i) The declarations that are made locally in the procedure.

(ii) The names of all enclosing procedures

(iii) The declarations of names made immediately within such procedures.

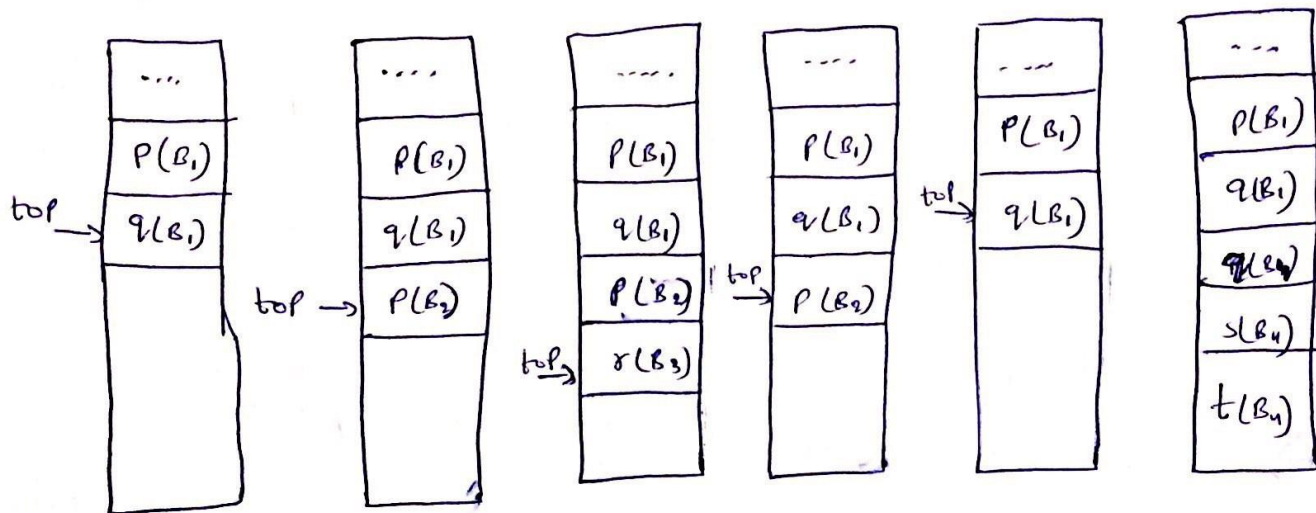
Eg:-

Scope-test (c)



→ The storage <sup>can be</sup> allocated for a complete procedure body at one time.

→ The storage for the names corresponding to particular block can be as shown below



### Lexical Scope for Nested Procedure

→ Nested procedure is a procedure that can be declared within another procedure.

→ A procedure P<sub>i</sub>, can call any procedure that is its direct ancestor or older siblings of its direct ancestor.

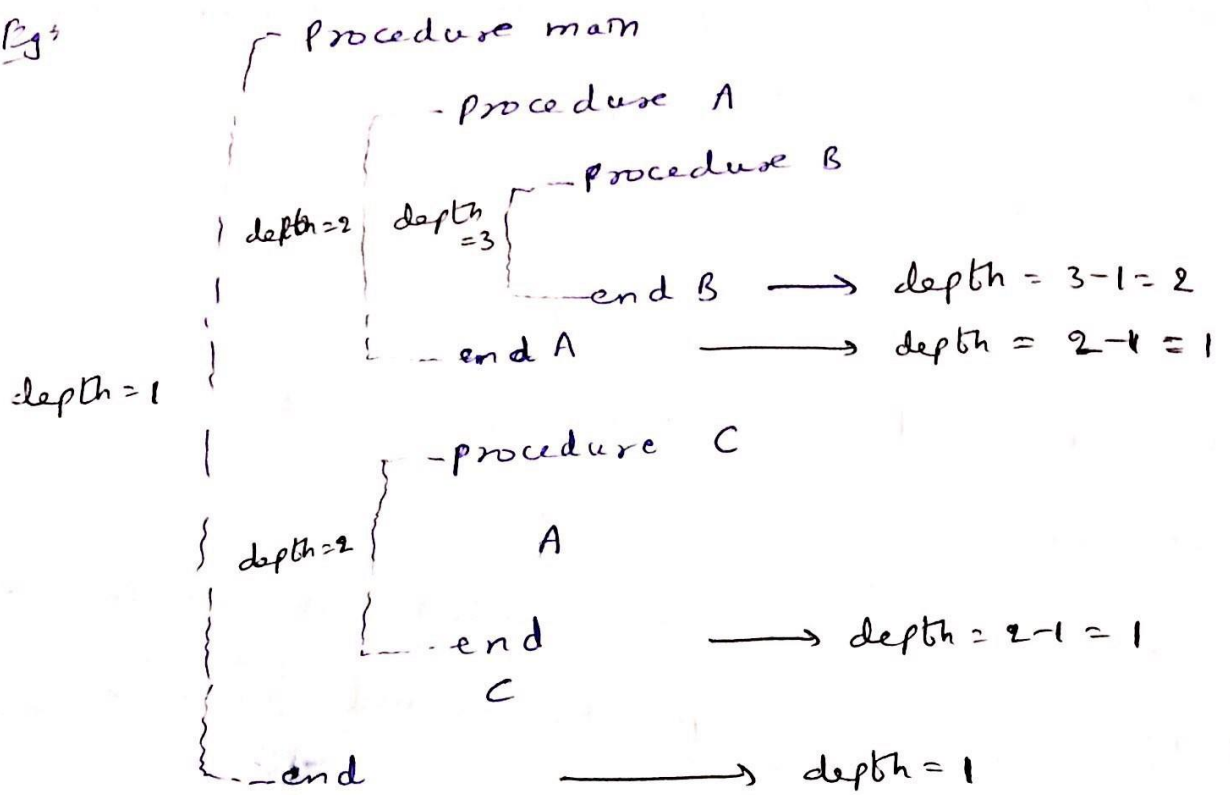
→ The nested procedures can be as shown

```

Procedure main
  Procedure P1
    " P2
    " P3
    " Pn
  
```



Ex 3



→ Nesting depth - Nesting depth of a procedure is used to implement lexical scope. The Nesting depth can be calculated as follows:

- (i) The nesting depth of main program is 1
- (ii) Add 1 to depth each time when a new procedure begins
- (iii) Subtract 1 from depth each time, when you exit from a nesting procedure.
- (iv) The variable declared in specific procedure is associated with nesting depth.

→ The lexical scope can be implemented using

Access links and Displays

Access link

→ The implementation of lexical scope can be obtained by using pointers to each activation record.

→ These pointers are called Access links.

✱ If a procedure P is nested within a procedure Q then access link of P points to access link of most recent activation record of procedure Q.

Eg: Program test;

```
var a: int;
```

```
procedure A;
```

```
  var d: int;
```

```
    begin a := 1, end;
```

```
  procedure B(i: int);
```

```
    var var b: int;
```

```
      procedure C;
```

```
        var k: int;
```

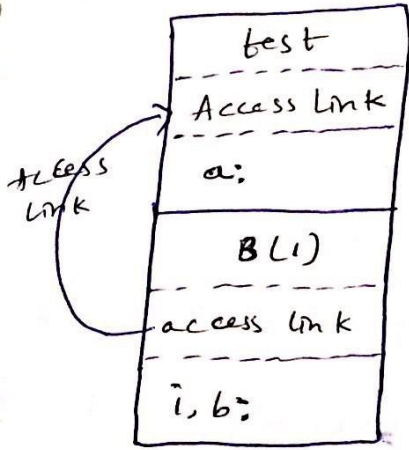
```
          begin A; end;
```

```
        begin
```

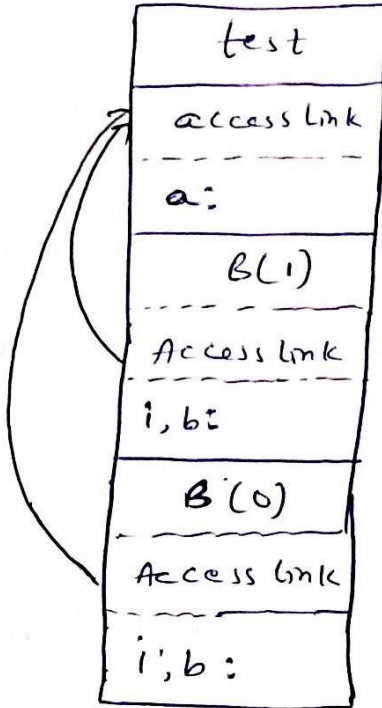
```

if (i < 0) then B(i-1)
else c;
end
begin B(i); end;

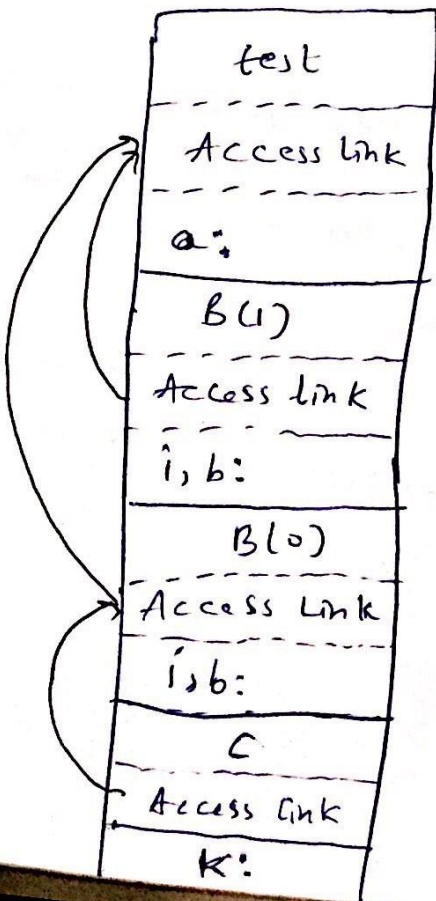
```



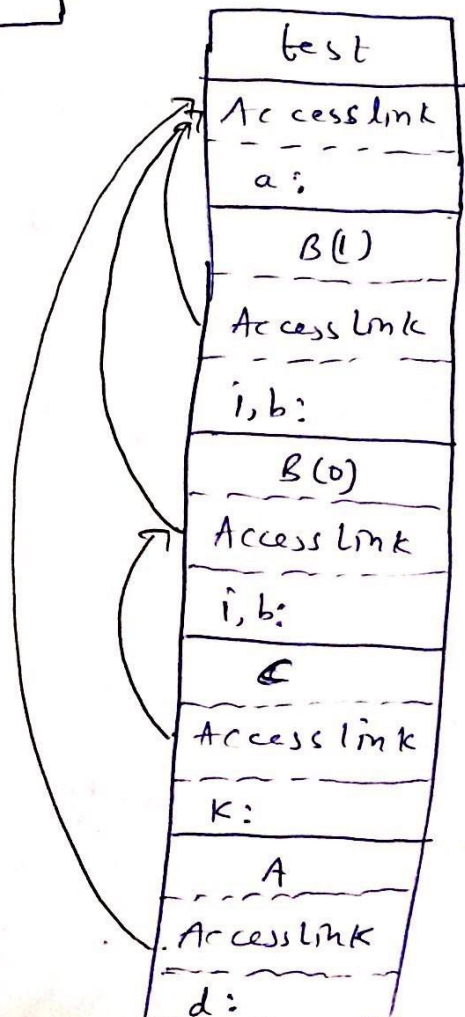
(a)



(b)



(c)





→ To set up the access links at compile time:

(i) if procedure A at depth  $n_A$  calls procedure B at depth  $n_B$  then

case 1: if  $n_A < n_B$ , then B is enclosed in A and  $n_A = n_B - 1$

case 2: if  $n_A \geq n_B$ , then it is either a recursive call or calling a previously declared procedure.

(ii) The access link of activation record of procedure A points to activation record of procedure B where the procedure B has procedure A nested within it.

(iii) The activation record for B must be active at the time of pointing.

(iv) If there are several activation records for B then the most recent activation record ~~with~~ <sup>will</sup> be pointed.

→ Thus, by traversing the access links, non locals can be correctly accessed.

## A Basic Mark and sweep Collector

→ Mark-and-sweep garbage collection algorithms are straight forward, stop-the-world algorithms that find all the unreachable objects, and put them on the list free space

→ Algorithm ~~marks~~ "visits" and "marks" all reachable objects in the first tracing step.

→ "Sweeps" the entire heap to free up unreachable objects.

Algorithm; Mark-and-sweep garbage collection.

Input: ~~The~~ A Root set of objects, A heap, and a freelist, called Free, with all unallocated chunks of the heap.

All chunks of space are marked with boundary tags to indicate their free/used status and size.

Output: A modified Free list after all the garbage has been removed.



## METHOD:

- The algorithm uses several simple data structures.
- List Free holds objects known to be free.
- A list called Unscanned, holds objects that we have determined are reached, but whose successors we have not yet considered. i.e., we have not ~~scanned~~ scanned these objects to see what other objects can be reached through them.
- The Unscanned list is empty initially.
- ~~Ad~~ Additionally, each object includes a bit to indicate whether it has been reached (the reached bit).
- Before the algorithm begins, ~~the~~ all allocated objects have the reached bit '0'.



## Marking Phase

- 1) set the reached bit to 1 and add to list unscanned each object referenced by the root set;
- 2) while (unscanned  $\neq \emptyset$ ) {
- 3) remove some object o from unscanned;
- 4) for (each object o' referenced in o) {
- 5) if (o' is unscanned i.e. its reached bit is 0) {
- 6) ~~set~~ set the reached bit ~~to~~ of o' to 1;
- 7) put o' in unscanned;
- 8) }
- 9) }
- 10) }

## Sweeping phase

- 11) Free =  $\phi$
- 12) for (each chunk of memory  $o$  in heap)  $\{$
- 13) if ( $o$  is unreached, i.e. its reached bit is  $0$ )  
add  $o$  to Free;
- 14) else set reached bit of  $o$  to  $1$
- 15) }

→ In line (1) of Marking phase, we initialize unscanned list by placing these all the objects referenced by the root set.

→ The reached bit ~~of~~ for all these object is also set to 1.

→ ~~From~~ Lines (2) to (7) are in loop, in which we, in turn, examine each object  $o$  this is ever placed in unscanned list.

→ The for-loop of lines (4) to (7) implements scanning of objects  $o$ .

→ we examine each object  $o'$  for which we find a reference within  $o$ .

→ If  $o'$  has already been reached (if reached bit is 1), then there is no need to do

anything about  $o'$ ; it has either been scanned previously, or it is not on unscanned list to be scanned later.

→ If  $o'$  was not reached already, then we need to set its reached bit to 1

in line (6) and add  $o'$  to the unscanned

list in line (7).

→ Lines (11) to (14), the sweeping phase, reclaim the space of all the objects that remain unreached at the end of the marking phase.



- Note that these will include any objects that were on the Free List originally.
- Because the set of ~~unreached~~ <sup>unreached</sup> objects cannot be enumerated directly, the ~~algorithm~~ algorithm sweeps through entire ~~heap~~ heap.
- Line (13) puts free and unreached objects on the Free List, one at a time.
- Line (14) handles the reachable objects by set their reached bit to zero (0), in order to maintain the proper predictions for the next execution of the garbage collection Algorithm.

## unit-8

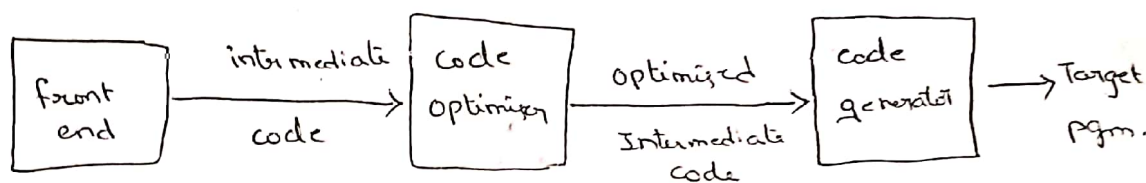
### Object code forms:-

The final phase in compiler design is code generation.

The i/p is optimized intermediate code of the i/p

source pgm & out <sup>put</sup> <sup>from</sup> of CG (code generator) is a

equivalent target pgm.



The o/p of the code generator is Target Pgm & object code.

It may be in different forms. Like

① Absolute machine language :- This code can be placed in a fixed memory location and immediately executed.

Ex:- Example of compiler which produce target code in the Absolute machine language are "WATFIV" & PL/C.

② Relocatable machine code :- In this form, we can allow subprogram to be compiled separately.

But we need to link together all such separately compiled sub-programs by linking and we need to load them for execution, for this we need loader.

If the target machine, can't able to do all this automatically, the compiler provide explicit relocation information to the loader to link the separately compiled sub-programs.

③ Assembly code is Assembly language code :-

If target code is in the form assembly-language code which contain symbolic instructions & use macro facilities. makes code-generation easier.

If the o/p is Assembly-language code

This type of o/p is applicable for target machine with small memory.

Machine-Dependent Code Optimisation :-

The code target code generated by the code generator is optimised in order to improve speed & size reduce the amount of memory required to execute.

So, the code optimisation is based on the machine in which code is going to execute.

It depends on the Target machine

1. Memory Management
2. Instruction Selection
3. Register Allocation.
4. Evaluation order.



## ① Memory Management

In front end & code generation phase, the names in the three-code statements are mapped to addresses of data objects in run-time.

The name in the TAC refers to the symbol-table entry for the name.

The type in the declaration determines the width - means the amount of memory required to store.

Based on that a relative address is determined for the name.

It is done in two ways

1. static Allocation
2. stack Allocation.

The information that is needed during the execution of a procedure is kept in a block of storage ~~structure~~ <sup>of storage</sup> called

"Activation Record".

In static Allocation, a fixed memory is allocated for Activation Record at compile time.

(a). The position of an activation record is fixed at compile time.

whereas in stack Allocation, the position of the activation record is not known until run time.

...andq ...

### Instruction Selection

The nature of instruction set of the target machine determine the difficulty in selecting the instruction.

Instruction speed & idioms are impoirt factors in the selection process.

Ex:- if we have exp  $a := b + c$ . is converted into code like

```
MOV b, R0.
```

```
ADD c, R0.
```

```
MOV R0, a
```

If we convert statement-by-statement into code, it produce code which is poor & inefficient

Ex:-

```
a := b + c
```

```
d := a + e.
```

```
MOV b, R0
```

```
ADD c, R0
```

```
MOV R0, a.
```

```
MOV a, R0.
```

```
ADD e, R0.
```

```
MOV R0, d.
```

→ In this case, we are moving R0 value to 'a' and once again we are moving a value to R0.

which is reduntant value, this is due to Statement-by-Statement code generation.

The quality of the code is determined by its speed & size. If # of instructions are less, then the size is reduced & speed is increased.

If target machine has such instruction set then we can implement a operation in many ways.

If we use efficient instruction set, then the instruction cost also reduced.

Ex:-  
 $a := a + 1 ;$   
MOV a, R0  
ADD #1, R0  
MOV R0, a.

If the target machine has instruction like "INC" we can perform this  $a := a + 1$  in 1 step like.

INC a, 1.

### Register Allocation :-

"Register" are one of the most important resource of the machine. Efficient utilization of resource of registers produce efficient target code.

The operands of the instructions are stored either at Register or memory.  
Instruction with register operands are short & fast when compare with instruction with memory operands.



If the number of variables is higher than the no of registers available, then the code generator should identify which variables are stored in registers & which variables are stored in memory.

Register Allocation : It is the process of identifying the <sup>what</sup> set of variables need to be retained in registers.

Register Alignment :- The process of alliging a specific register to a specific variable.

Choice of Evaluation order :- The order in which the instructions are performed also can effect the efficiency of the target code.

Addressing Modes :-

- MOV            move source to destination.
- ADD            ADD source to Destination.
- SUB            Subtract source from Destination.

<u>MODE</u>	<u>FORM</u>	<u>Address</u>	<u>Added cost</u>
Absolute	M	M	1
Register	R	R	0
indexed	C(R)	C + contents(R)	1
Indirect register	*R	contents(R)	0
Indirect indexed	*C(R)	contents(C + contents(R))	1
Literal	#C	C	1

Ex: ①  $\text{MOV } R_0, M$

store the contents of Register  $R_0$  into memory location  $M$ .

②  $\text{MOV } \#4(R_0), M$

store contents  $(4 + \text{contents}(R_0))$  into memory location  $M$ .

③  $\text{MOV } \#5, R_0$

store constant 5 into Register  $R_0$ .

### Instruction Cost :-

The cost of an instruction be one plus the cost associated with the source & destination address.

The instruction cost corresponds to the length of the instruction.

The Addressing mode involve Registers have cost - 0

" " " " memory of literal " - 1  
locations

In most machines, but most instructions, the time taken to fetch an instruction from memory is more when compared the time taken for executing it.

Ex:-  $a := b + c$

$\text{MOV } b, R_0$  -  $\bullet + 1 + 0 = 2$

$\text{ADD } c, R_0$  -  $\bullet + 1 + 0 = 2$

$\text{MOV } R_0, a$  -  $\bullet + 0 + 1 = 2$

6

$$\begin{array}{rcl}
 \text{MOV } b, a & : & 1+1+1 & : & 3 \\
 \text{ADD } c, a & : & 1+1+1 & : & 3 \\
 \hline
 \text{cost} & : & & & 6.
 \end{array}$$

Assume that if  $R_0, R_1, R_2$  contain  $a, b, c$  address, then

$$\begin{array}{rcl}
 \text{MOV } *R_1, *R_0 & : & 0+0+1 & : & 1 \\
 \text{ADD } *R_2, *R_0 & : & 0+0+1 & : & 1 \\
 \hline
 \text{cost} & : & & & 2.
 \end{array}$$

② If  $R_1, R_2$  contain the values of  $b$  &  $c$  respectively.

And 'b' is not required further then

$$\begin{array}{rcl}
 \text{ADD } R_2, R_1 & : & 0+0+1 & : & 1 \\
 \text{MOV } R_1, a & : & 0+1+1 & : & 2 \\
 \hline
 \text{cost} & : & & & 3
 \end{array}$$

For generating good code, we need to utilize addressing capabilities efficiently, by which we can reduce instruction cost.



## DAG Representation of a Basic Blocks :-

Directed Acyclic graph, is a data structure which is used for implementing transformations on basic blocks.

A DAG for basic blocks is constructed with the following labels on nodes.

1. Leaves are labelled by unique identifiers, variable names are constants. It is subscript with 0, ... to avoid confusion with labels denoting "current values" of the identifiers.
2. Interior nodes are labeled with operator symbol.
3. Nodes are <sup>labelled with</sup> <sub>given</sub> sequence of identifiers. It specifies that this identifiers are holding that computed values.

### DAG Construction :-

In this process, each TAC statement is processed. If it come across a stmt like  $x := y \text{ OP } z$ . First it check, whether there is node for current value of 'y' exist already, if so uses it. Otherwise it creates a new node and label it as 'y'.

The same is done for 'z'. Also. we have two types of operators

Class 1 : ADD, SUB, MUL, DIV, UMINUS, ADDR-OF, ASSIGN, R-INDEX ASSIGN.

CLASS 2 - LT, GT, LE, GE, EQ, NE, L-INDEX ASSIGN, PROC-BEGIN, PROC-END, RETURN, CALL, LBL, GOTO etc.

If OP is one of class 1 operators,

Then find any internal node "P" with child nodes as 'y' & 'z'. If we don't find such a node create a new node labelled as 'x' with child nodes 'y' & 'z'.

② If we find, then add the identifier 'x' to the list of identifiers attached to 'P'.

① If 'x' is attached to some other dag node previously, remove that link, Associate the current value of 'x' with 'P'.

if the TAC is of form  $x := y$ .

If 'x' is attached to some other dag node previously, remove that link, associate the current value of 'x' with 'y'. Add the 'x' to the list of identifiers attached to 'y'.

Ex:-

```
int main ( )
```

```
{
```

```
  a = (b+c) * d ;
```

```
  e = f * a ;
```

```
  t = (b+c) * e ;
```

```
  g = (b+c) / d ;
```

```
}
```

```
(1) proc - begin main
```

```
  t0 := b+c.
```

```
  t1 := t0 * d .
```

```
  a := t1
```

```
  t2 := f * a .
```

```
  e := t2 .
```

```
  t3 := b+c . b * t1
```

```
  t4 := t3 * e
```

```
  t = t4 .
```

```
  t5 := b+c
```

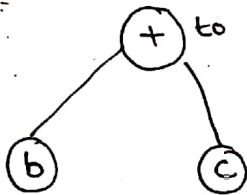
```
  t6 := t5 / d .
```

```
  g := t6 .
```

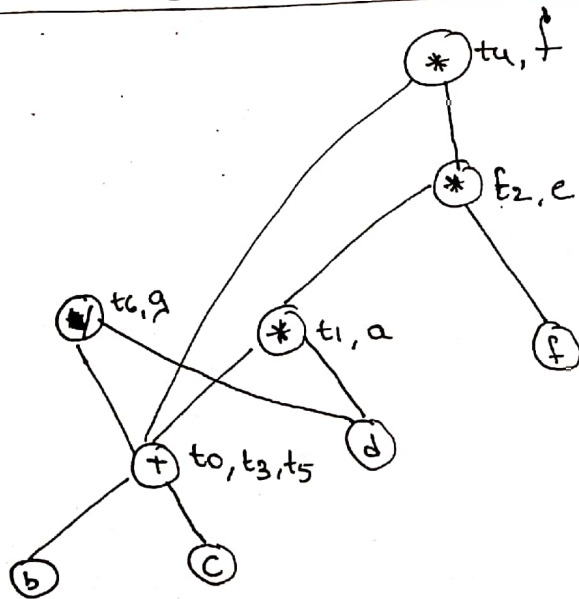
```
proc - End .
```

DAG :-

①



②





Optimization of Intermediate code by DAG :-

(21)

Reconstruction of Intermediate code by DAG :-

(21)

Application of DAG :-

→ Optimizing transformations like common sub-expression elimination, dead store elimination, copy propagation etc occur during DAG construction.

Whenever we are creating a new node, if we check whether previously a node is exist with same children & with same operator.

If not then only we are creating new node. This process allow us to detect common sub-expression & eliminate us from recomputing common-sub-expression.

Ex:- for Statement

$t_3 := b + c.$

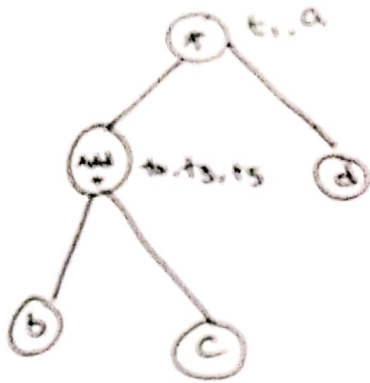
Already node 't' is exist, with same exp  $b+c$ , so we avoid re-computing 'b+c', & simply add identifier  $t_3$  is to like that for  $t_3 := b+c$ . also.

If we a node have multiple identifiers as labels we select ~~one~~ any one among them to represent the value. This eliminates unnecessary assignment stmt

of the form  $a := b$

Like this we implement copy propagation.

Ex:



we take only 'a' out of t1, a,  
since 'a' is a user-defined variable.

we replace

t1 := t0 \* d ;

a := t1 ;

## A simple code generator :-

A code generator generate target code for a sequence of TAC statements.

→ Ex:- If there is a TAC statement like this

$$x := y \text{ OP } z. \quad \rightarrow \quad x := y + z ;$$

First it check the operands  $y, z$  are in registers.

If they are in registers, it use them directly.

If then find what is Target language operator

for '+'.

If the Target language is Assembly, then for '+'

it find it is "ADD".

Then it compute the statement and store the result in the register, until that register is needed for any other computation.

The code generator generate target code in different

ways

① If  $y$  is in Register -  $R_0$  &  $y$  is not used in any other statements.

If  $z$  is in Register -  $R_1$  but  $z$  is going to use in other statements.

Then it generate code this

ADD  $R_1, R_0$  -

$R_1$  -  $z$ .

$R_0$  -  $y$ .

$z+y$  = value in  $R_0$ .

value is in  $R_0$ .



② If Y is in Register R<sub>0</sub> and Y is not used next.  
Z is in memory location, Z is used next.

Then it generate code like this.

Add Z, R<sub>0</sub>

↓

Here Z is the memory location of variable 'z'.

③ If Y is in R<sub>0</sub> & Y is not having any next use  
Z is in memory location Z but it is used frequently  
in the remaining code, then it creates code like this

Mov Z, R<sub>1</sub> ;

ADD R<sub>1</sub>, R<sub>0</sub> ;

It is fast to fetch Z value next time as it is stored  
in Register, instead of memory location.

For doing this, it uses 2 Descriptors of 2 Data Structures

① Register Descriptor : It keep track of what is currently  
in each Register. (RD)

It give information about which variables are currently  
held in a particular Register.

Initially RD show all Registers are empty. As code  
generator proceeds, each register will hold zero or more variables.  
at any given time.

If the statement is copy statement, then we use single register to hold more than one variable.

Ex:-  $x = y$ .  $a = b$   
 If  $x$  is in  $R_0$ . then  $R_0$  hold even 'y' also.

(21)  
 we can store both  $x$  &  $y$  variables in single register.

Register Name	Current variables
$R_0$	$a, y$
$R_1$	$y$
$R_2$	$z$
	⋮

→  $a = b$   
→

Address Descriptor :- It keep track of the location of locations where the current value of the variable can be found at run time. (22)

The location might be a register, memory location, stack location or some set of these.

Ex:-

Variable name	Current location
$y$	$R_0$
$z$	Memory location $z$ .
$x$	$R_0, ML-x$
⋮	⋮

## Code Generation Algorithm :-

\* IIP is TAC Statement OP is Target language code.

Ex: If IIP  $x := y \text{ OP } z$ .

Step 1 :- It first invoke getreg function.

This function return the location 'L' where the result of the computation  $x := y \text{ OP } z$  is stored.

→ 'L' may be a register or memory location.

Ex: ① If  $y$  is in  $R_0$ , &  $y$  is no longer going to use.

Then it return ' $R_0$ ' as 'L'.

② If  $y$  is going to use in other statements, then it check the list of available registers, then send one of the Register as 'L'.

→ If case 1 fail we are going to case 2.

If case 2 fail means all registers are occupied with some variables,

③ Then it send select some available memory location  $m$  as send 'm' value as 'L'.

Step 2 :- Then it consult Address Descriptor, to current location of  $y$ . If it is in both  $R$  &  $m$  then it select  $R$ .

If it is in only  $m$  then it perform the following stmt

MOV  $y, L$ .



step 3. consult the AD to get current location of z.  
If it is in both R & M select R. and perform

OP R, L.

↓  
z is in Register R.

If it is memory location z then it performs like this

OP Z, L.

③ Then MOV x, L. update the Address Descriptor of x as 'L'.

If 'L' is Register, update its RD with 'x'.

④ If y, z are in Register R<sub>1</sub>, R<sub>2</sub> & they are no longer

used in the block after this statement.

free the registers R<sub>1</sub>, R<sub>2</sub>.

Ex:-

## Register Allocation & Assignment Techniques:

(Register Allocation is the process of identifying what variables need to be stored in the registers.)

In one method, some set of registers are assigned to store base addresses, some set to store stack-pointers, some to store the result of arithmetic computations.

This method make compiler design easy, but strict use of method leads to wastage of <sup>register</sup> resources. For efficient utilisation of registers some Register Allocation techniques are there.

### 1. Global Register Allocation:

If Register allocation is performed at a basic block level is called "Local Register Allocation".

All the variables that are live at the end of the basic block which are present in registers are spilled & saved into the memory location. So that the successor blocks can generate code correctly.

But for loops, in LRA, spilling should done frequently. So instead of LRA, In Global register Allocation - registers are allocated across the basic blocks.

In this, registers are allocated to frequently used variables throughout a loop.

In x86 language, by using "Register" storage class directly assign a register to a variable.

Usage counts :-

If 'x' is a variable present in register r.

Then each reference to 'x' save one unit of cost.

To measure the frequency of usage of a variable we have a metric of one unit of cost being saved for each use of this variable.

Ex:- If 'x' is live at the exit of the loop & used outside. we save two units of cost. <sup>basic block</sup>

v. r. Over ~~write~~ by avoiding store & load operations.

\* If 'x' is used at the entry of loop, ~~first~~ it must be loaded before getting into the loop which require two units of cost.

So, for each exit block of loop at which 'x' is live on entry to some successor of B outside of the loop, it is stored at the cost of two units.



so, the advantage of allocating a register to  $x$  within the loop  $L$  is given by

$$\sum_{\text{Block } B \text{ in } L} (\text{use}(x, B)) + 2 * \text{live}(x, B).$$

$\text{use}(x, B)$  is the number of times  $x$  is used in  $B$  prior to any definition of  $x$ .

$\rightarrow \text{live}(x, B) = 1$ , if it is live on exit from  $B$  and assigned a value in  $B$ .

$\rightarrow \text{live}(x, B) = 0$ , if it is not live on exit from  $B$ .

### Graph colouring :-

It is a systematic technique for allocating registers & managing register spills. In this method, we use two passes are used.

1. In the first pass, target machine instructions are selected with names used in the intermediate code became name of the registers & symbolic registers.

once the instructions selected, second pass starts. In the second pass, physical registers are assigned to the symbolic ones. and also a register-interference graph is constructed for each block.

In this nodes are symbolic registers, an edge present b/w two nodes, if one is live at a point where the other is defined.

Ex:  $\begin{matrix} 1. & a = b + c \\ 2. & d = d - b. \end{matrix}$

→ Registers are a, b, c

Registers are d, b.

In Register-interference graph nodes are a, b, c, d.

An edge is present b/w this nodes if one is live at a point where the other is defined.

At line '2' 'a' is live & 'd' is defined. So

we connect node 'a' & node 'd' with an edge.

Here In this method the Register-interference graph is colored with k-colors, where 'k' is the no of assignable registers.

The graph is colored like that no two adjacent nodes have same color.

Here color means register. it implies that no two names are assigned with same physical register.

Code generation by DAG:

1. Rearranging the order:

to i/p stmt :  $(a+b) - (c - (c+d))$ .

Then TAC statements are

$$t_1 := a + b$$

$$t_2 := c + d$$

$$t_3 := c - t_2$$

$$t_4 := t_1 - t_3$$

code generation by DAG is done in three ways:-

1. Rearranging the order:-

The order in which the TAC statements are executing will affect the cost of the target code generated. If we rearrange, means if we change the order in which TAC statements execute we can optimise the target code.

Ex:-  $a + b + (c - (c + d))$ .

①  
 $t_1 := a + b$   
 $t_2 := c + d$   
 $t_3 := c - t_2$   
 $t_4 := t_3 + t_1$

~~object code~~

TAC

③  
 $t_2 := c + d$   
 $t_3 := c - t_2$   
 $t_1 := a + b$   
 $t_4 := t_3 + t_1$

object-code

④  
 1. mov c, R0  
 2. ADD d, R0  
 3. MOV e, R1  
 4. SUB R0, R0  
 5. mov a, R0  
 6. ADD b, R0  
 7. ADD R1, R0  
 8. mov R0, t4

② R0 & R1 are available Register, each statement is converted into object code. Means line-by-line code is converted into Target obj. code.

object-form.

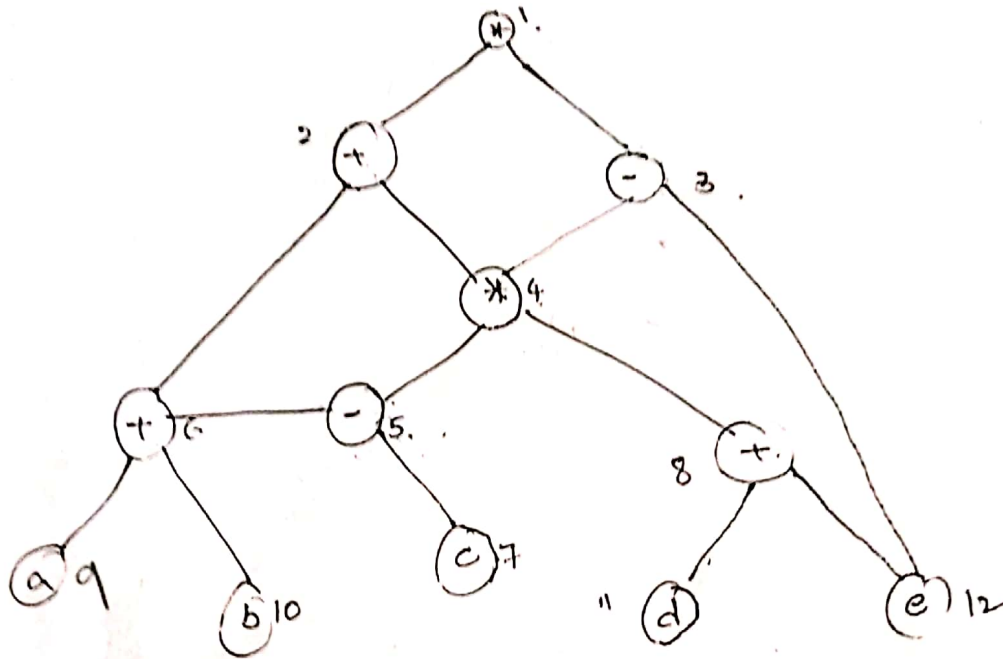
① MOV a, R0  
 2. ADD b, R0  
 3. MOV c, R1  
 4. ADD d, R1  
 5. MOV R0, t1 -  $t_1 := a + b$   
 6. MOV c, R0  
 7. SUB R0, R1 -  $c - (c + d)$   
 8. MOV t1, R0  
 9. ADD R0, R1 -  $R_0 := t_1 + b$   
 10. MOV R1, t4



② Heuristic Ordering

In this ordering, the evaluation of a node immediately follow the evaluation of its leftmost argument.

Algorithm :-



unlisted :-

~~1 2 3~~

unlisted

List

initially - (1)

1

(1)

left-child of 1 is 2, move 2 to list

1 2

left-child of 2 is 6, but it has one more parent '5' which is not in list.

So, we move to 3, its parent is '1' which is in list so we move 3 to list.

1 2 3

'3' left child is 4, it has  
 no parents 2 & 3 both are  
 in lists so move 4 to  
 list.

1 2 3 4

4 left child 5, move 5 also to list

1 2 3 4 5

5 " " '6' it has two parents

1 2 3 4 5 6

'5' & '2' both are in list, so  
 move '6' also to list

'6' left child is '9' but it

is internal node, so we move

1 2 3 4 5 6 8

to the right child of '4'

which is 8. Its parent is '4'

so we move '8' to list.

8 left child is '11' which is internal node now we stop.

→ The list is 1 2 3 4 5 6 8. The order of evaluation  
 is in reverse order of the list i.e., 8 6 5 4 3 2 1.

$$\begin{aligned}
 t_8 &= d + e \\
 t_6 &= a + b \\
 t_5 &= t_6 - c \\
 t_4 &= t_5 * t_8 \\
 t_3 &= t_4 - e \\
 t_2 &= t_6 + t_8 \\
 t_1 &= t_2 * t_3
 \end{aligned}$$

### ③ Labelling Algorithm

It has two parts.

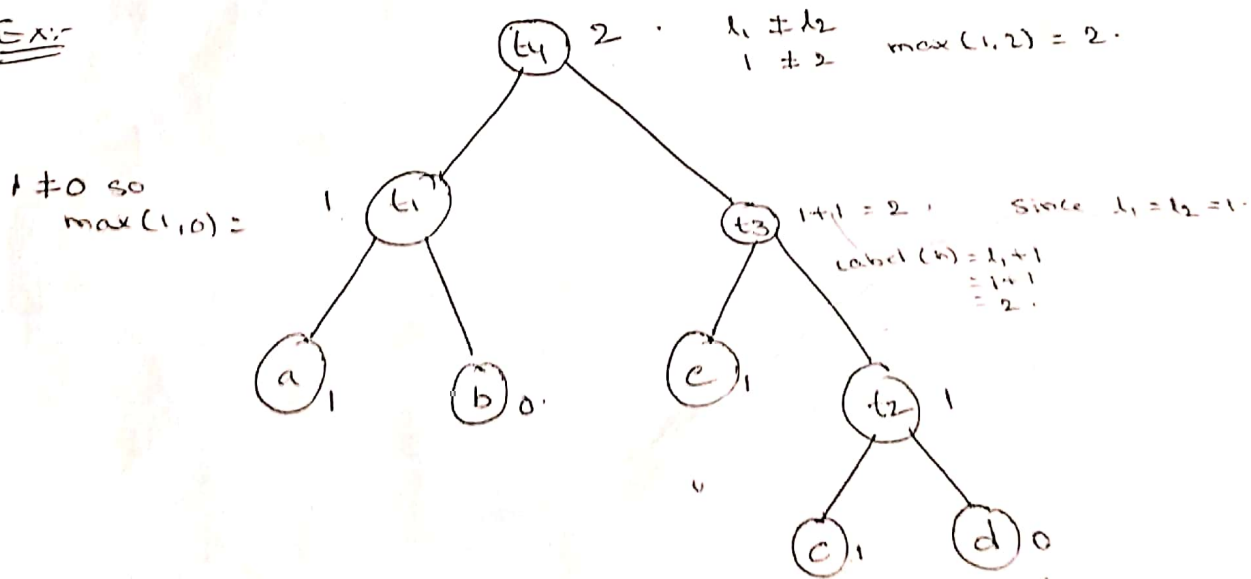
1. In first part, in bottom-up fashion label each node with an integer, without store of the intermediate values. So few registers are required.
2. In the second part, tree traversal is done. The order is based on the computed node labels.

Labeling is done like that, a node is not visited until all its children are labeled.

It is given by

$$\begin{aligned} \text{Label}(n) &= \max(l_1, l_2) && \text{if } l_1 \neq l_2 \\ &= l_1 + 1 && \text{if } l_1 = l_2 \end{aligned}$$

Ex:-



If 'n' is leaf node and it is left leaf node of its parent

Label(n) = 1 if it is right leaf node leaf(n) = 0.



## Peephole optimization:

If we produce target code by converting the statement by statement, we get redundant instructions.

It can be optimized & improved by applying transformations on the target Pgm.

"Peephole optimization" is one of the simple & effective optimization techniques which is done locally to improve target code.

[It is done on local on a small segment of code called peephole & window.]

It is done directly after intermediate code generation to improve the intermediate representation instead after code generation.

The peep-hole code need to be continuous code.

The optimization techniques that can be done on

peep-hole are peep-hole optimization techniques :-

→ First we find peep-hole instructions & replacing them with fast & shift instructions.

1. Redundant - instruction elimination
2. flow - of - control optimisation
3. Algebraic simplifications
4. Use of machine idioms.

### ① Redundant instruction elimination :-

(a)

Redundant load & store :-

Ex:- If we have two instructions like this

```
① MOV R0, a
② MOV R0, a
MOV R0, a
MOV a, R0.
```

We remove instruction ② specifying that value of 'a' already in 'R0' register.

### ② Removing unreachable code :-

Ex:- if age > 18

# define debug 0.

```
if (debug)
{
=
}
```

Intermediate code

If debug = 1 Goto L1  
Goto L2.

L1 : debugging info

L2 : - - - -

It is replaced like

```
if debug != 1 Goto L2
print debug info.
```

L2 : - - -

It remove jump over jumps.

By copy propagation it is converted as

if  $0 \neq 1$  goto L2.  
print debugging inf

L2:

Use of machine Idioms:-

Some Target machines may have different hardware instructions to implement certain specific operations efficiently.

If we use such instructions we can reduce execution time & cost significantly.

Ex:- Instructions are present for Auto increment.

Auto decrement etc:-

$a = a + 1$   
mov a, R0  
ADD #1, R0  
mov R0, a.

INC i, 1  
mov i, R0.

9/07/16 Absentees 2nd

1, 2, 4, 6, 10, 11, 13, 16, 18, 19, 20, 22, 23, 24,  
25, 28, ~~30~~, 31, 32, 33, 34, 36, 37, 38, 41, 42, 43,  
44, 45, 46, 48, 49, 50, 51, 52, 54, 55, 57, ~~58~~

L2, L3, 4, 5, 8, 6 57 52 4. ~~58~~ ~~59~~

28/7/16

5th hour

Abstract Lab

5, 6, 11, 13, ~~14~~, 24, 27, 28, 31, 35, 37, 39, 49, 50, 51, 53, 57, 58, 59  
60. LE-3, RE-524, 561, 512.



Code Optimisation :- The process of improving the intermediate code in terms of speed & amount of memory required for execution is called "Optimisation".

Optimisation Techniques :-

Constant Folding :- In this, the constant expressions in the input source prgm are calculated and replaced by the equivalent values at the time of compilation.

Ex:-

code

P = a [3]

a [3] = P

Normal Intermediate code

t0 := 3 \* 4  
t1 := 2 \* a  
P := t1  
t2 := t1 [t0]  
P := t2

t0 := 3 \* 4  
t1 := 2 \* a  
t1 [t0] := P

Optimised Intermediate code

t0 := 12  
t1 := 2 \* a  
t2 := t1 [t0]  
P := t2

t0 := 12  
t1 := 2 \* a  
t1 [t0] := P

copy propagation & Dead code Elimination :-

Ex:-

If a = b. the use of variable 'b' instead of variable 'a'.

```
P = a[3]
```

```
t0 := 12
t1 := &a
t2 := t1[t0]
P := t2.
```

```
t1 := &a
t1[3] := a
t2 := t1[12]
P := t2.
```

```
a[3] = 10;
```

```
t0 := 12
t1 := &a
t1[t0] := 10.
```

```
t1 := &a
t1[12] := 10.
```

It is also called as copy propagation.

Dead code Elimination :- ~~Code~~

code that is never executed by the prgm is called Dead code.

Eliminating dead code is called Dead code Elimination.

This technique reduces the memory required for the prgm.

Ex:-

~~int~~ ~~vote~~

```
int vote; void main()
{
  int age = 15, vote = 0;
```

```
  if (age > 18)
    vote = vote + 1;
}
```



Intermediate code

```

proc - begin main
  age := 15
  If 15 > 18 goto L0
  goto L1
Label L0
  to := vote + 1
  vote := to
Label L1
proc - end main.

```

```

proc - begin main
  age := 15
  goto L1
Label L1
proc - end main.

```

IC after copy propagation

After dead code elimination

Algebraic Transformations ::

Name of the Identity

Additive Identity

Multiplicative Identity

Multiplication with 0

Example

$x + 0 = x$

$x * 1 = x$

$x * 0 = 0$

IC Statement

=

$y := x + 0$

$y := x * 1$

$y := x * 0$

IC after Transformation

=

$y := x$

$y := x$

$y := 0$

Name of the Identity =

Additive Identity

Multiplicative Identity

Multiplication with 0



### Strength Reduction Transformation:

Addition operation takes fewer cycles than multiplication operation.

Shift operation takes fewer cycles when compared to multiplication or division operation on most of the processors.

Strength Reduction Transformation identifies & replace costly operations by less expensive one, which will have the same effect.

Ex:-  $y_1 = x + 2$ ;  $y_1 = x + x$   
 $y_1 = x * 32$ ;  $y_1 = x \ll 5$

### Common Sub-expression Elimination:

If any expression or a part of a expression or sub-expression is present more number of times in a Pgm it called common sub-expression.

The process of identifying common sub-exp & eliminating their computation multiple times in the IC is known as

Ex:-  

```
int main()
{
  int a, b, c, s, avg;
  s = a + b + c;
  avg =  $\frac{a + b + c}{3}$ ;
}
```

```
proc - begin main
  t0 = a + b + c;
  s = t0;
  t1 = a + b + c;
  t2 = t1 / 3;
  avg = t2;
proc - end main.
```

Ex:-

```

int Sum(int n)
{
Sum-n = (n * (n+1))/2 ;
Sum-n1 = (n * (n+1) * (2 * n+1))/6 ;
}

```

```

proc -begin Sum
  goto Label Lo.
Label Lo.
  t0 := n+1
  t1 := n * t0
  t2 := t1/2
  sum-n := t2
  t3 := n+1
  t4 := 2 * t3
  t5 := 2 * n
  t6 := t5 + 1
  t7 := t4 * t6
  t8 := t7/6
  sum-n1 := t8
proc -end Sum.

```

IC After common sub-expression elimination:-

Here common sub-exp is :  $n * (n+1)$ .

```

proc -begin Sum
  t0 := n+1
  t1 := n * t0
  sum-n := t1/2
  t2 := 2 * n
  t3 := t2 + 1
  t4 := t1 * t3
  t5 sum-n1 := t4/6 ;
  sum-n1 := t5
proc -end Sum.

```

## Constant Propagation

- In this technique the value of variable is replaced & computation of an exp is done at the compile time.

```
ex pi = 3.14;  
r = 5;  
Area = pi * r * r
```

## Loop Optimizing Techniques :-

③ Loop Unrolling - No. of jumps & tests can be reduced by writing the code two times.

eg Copying contents of one array to another -

```
int i=0;  
while (i < 10) do  
{  
    b[i] = a[i];  
    i++;  
}
```

```
int i=0;  
while (i < 10)  
{  
    b[i] = a[i];  
    i++;  
    b[i] = a[i];  
    i++;  
}
```



## Loop optimisation :-

In general, the statements in a loop get executed many times, if we optimise the code in the loop, then we get good performance improvement.

### Loop optimisation Techniques :-

① Loop invariant Code Motion :- The statements within a loop whose values are computed, whose value do not change throughout the life of the loop are called "loop invariant statements". Such statements are identified & moved outside of the loop.

This technique is called "loop invariant code motion".

Ex:-

```
int func(int a, int b)
{
    int i, n1, n2;
    i = 0;
    n1 = a * b;
    n2 = a - b;
    while (i < (n1 * n2))
    {
        i = i + 1;
    }
    return (i);
}
```

```
proc - begin func
    i := 0
    n1 := a * b
    n2 := a - b.
    label .L0
    to := n1 * n2.
    if i < to goto .L1
    goto .L2.
Label .L1
    i := i + 1
    i := i
    goto .L0.
Label .L2
    return i.
proc - end func.
```

Then the code after loop invariant code motion is:

```
proc-begin func
```

```
  i := 0
```

```
  n1 := a * b
```

```
  n2 := a - b.
```

```
  to := n1 * n2.
```

```
  Label .L0
```

```
  .  
  .  
  .  
  .  
  .
```

```
proc-end func.
```

② Strength Reduction on induction variables :-

A variable that changes by a fixed quantity on each of the iterations of a loop is called as "Induction Variable".

```
int ind;
int a[10];
int func()
{
  while (ind < 20)
  {
    a[ind] = 10;
    ind = ind + 1;
  }
}
```

```
proc-begin func
  Label .L0
  if ind < 20 goto .L1
  goto .L2
  Label .L1
  to := ind * 4
  t1 := 2a
  t1[t1] := 10
  t2 := ind + 1
  ind := t2
  goto .L0
proc-end func.
```

```
proc-begin func
  to := ind * 4
  Label .L0
  if ind < 20 goto .L1
  .
  t1 := 2a
  t1[t1] := 10
  ind := ind + 1
  to := to + 4
  goto .L0
proc-end func.
```

$a(5)$   
 $to = 5 \times 4$   
 $i = 2a$

$a(5)$

0 1 2 3 4  
 $0+4$   $4+4$   $8+4$   $12+4$   $16+4$

## Basic Blocks:-

Alg: Partition into basic blocks

Input: A sequence of three address statements

Output: A list of basic blocks with each three-address statement in exactly one block.

Method:-

1) We first determine the set of leaders, the first statements of basic blocks.

The rules we use are the following:-

(i) The first statement is a leader

(ii) Any statement that is the target of a conditional or unconditional goto is a leader.

(iii) Any statement that immediately follows a goto or conditional goto statement is leader.

2) For each leader, its basic block consists of leaders and all statements up to but not including the next leader or end of program.

Ex:-

```
int func1(int c)
{
    v3 = v1 + v2;
    if (c > 100)
    {
        v4 = v1 + v2;
        v1 = 0;
    }
    v5 = v1 + v2;
}
```

```
proc-begin func1
    v3 = v1 + v2
    if (c > 100) goto L0
    goto L1
Label L0
    v4 = v1 + v2
    v1 = 0;
Label L1
    v5 = v1 + v2
Label L2
proc-end func1
```



proc begin func

2. int func (int a, int b, int c)

{

int x, y, z;

x = 2 \* a;

y = 2 \* a + 5 \* b;

if (a > 1)

{

x = 2 \* a + 3 \* b + 20;

y = 2 \* a + 4 \* b + 10;

}

z = x \* y;

return z;

}

proc begin func

t0 := 2 \* a

x := t0

t1 := 2 \* a

t2 := 5 \* b

t3 := t1 + t2

y := t3

if a > 1 goto Label L1

goto Label L1

Label L1

t4 := 2 \* a

t5 := 3 \* b

t6 := t4 + t5

t7 := t6 + 20

x := t7

t8 := 2 \* a

t9 := 4 \* b

t10 := t8 + t9

t11 := t6 + t10

y := t11

Label L1

t12 := x \* y

z := t12

return z

goto Label L2

Label L2

proc - end func

B0

B2

B3

B4

Local optimisation: performing transformation inside a basic block is called local optimisation.

```

Bo
proc-begin func1
  t0 := 2 * a
  x := t0
  t1 := 2 * a
  t2 := 5 * b
  t3 := t1 + t2
  y := t3
if a > 1 goto label .L0
  
```

Bo. optimised code.

```

proc-begin func1
  x := 2 * a
  t2 := 5 * b
  t3 := x + t2
  y := x + t2
  
```

Description  
 to = 2 \* a is eliminated by applying ~~constant~~ copy propagation  
 This is eliminated by copy propagation

if a > 1 goto label .L0.

B1. optimised code.

```

goto label .L1
  
```

```

B1
goto label .L1
  
```

B2. optimised code.

```

label .L0
  t4 := 2 * a
  t5 := 3 * b
  t6 := t4 + t5
  x := t6 + 20
  t9 := 4 * b
  t10 := t9 + t9
  y := t10 + t0
  
```

Description  
 The statement  $x := t6 + 20$  is ~~eliminated~~ using copy propagation.  
 $t8 := 2 * a$  is eliminated based on common-sub exp elimination

```

B2
label .L0
  t4 := 2 * a
  t5 := 3 * b
  t6 := t4 + t5
  t7 := t6 + 20
  x := t7
  t8 := 2 * a
  t9 := 4 * b
  t10 := t8 + t9
  t11 := t10 + t0
  y := t11
  
```

B3. optimised code

```

label .L1
  z := x * y
  return z
goto label .L2
  
```

Description  
 The statement  $t12 := x * y$  is eliminated by using copy propagation, then the  $z := x * y$  ;  
 No optimisation.

```

B3
label .L1
  t12 := x * y
  z := t12
  return z
goto label .L2
  
```

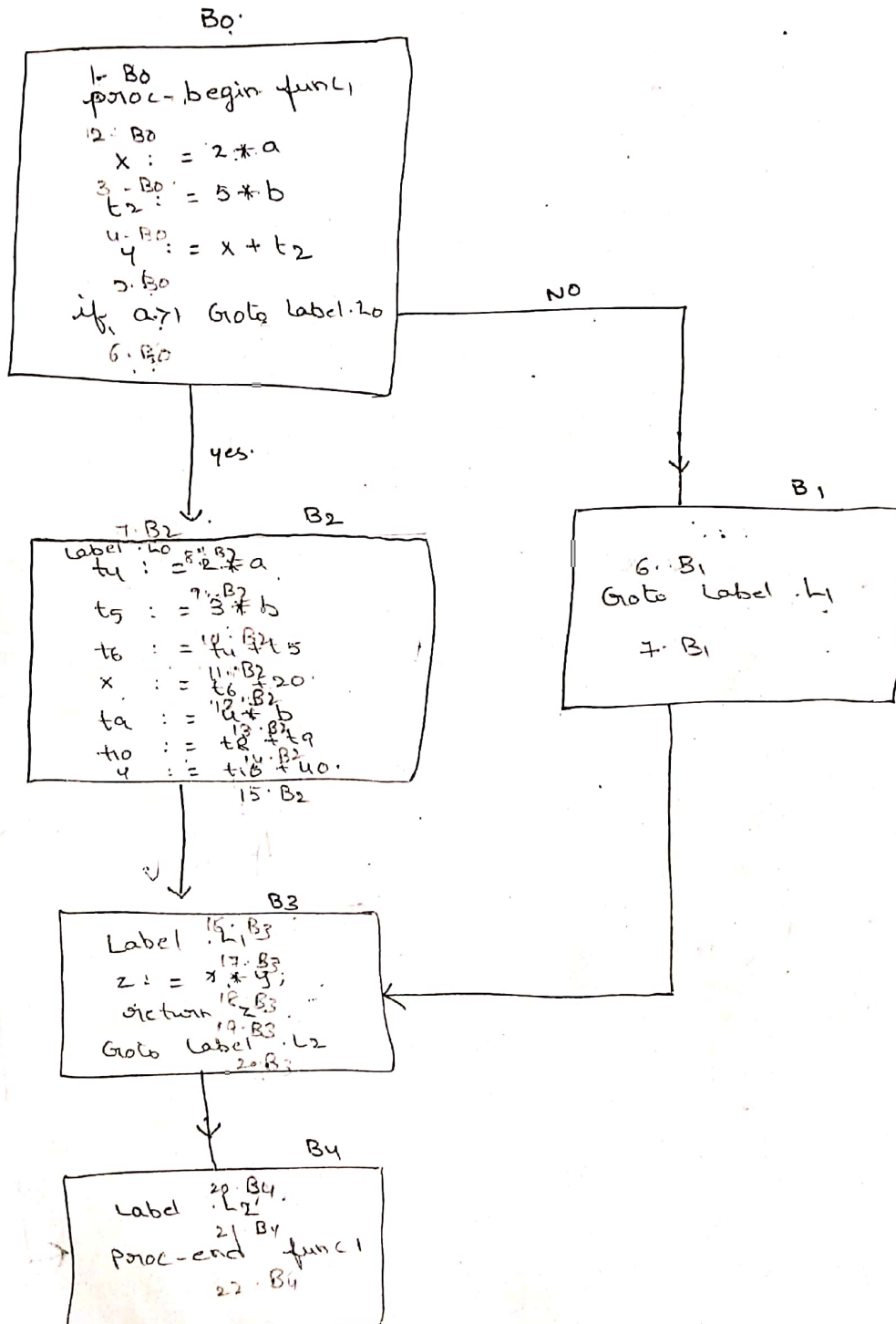
B4. optimised code  
 label .L2  
 proc-end func1

```

B4
label .L2
proc-end func1
  
```

Flow-Graph: The way at how the flow of control goes from one block to another block is represented in the form of a directed graph is called as "Flow-Graph".

Ex:-





→ The initial node

→ The starting node in a flow graph is called "Initial node" or the node which contains the first TAC statement as a leader is called "initial node".

In this example - B<sub>0</sub> is initial node.

B<sub>1</sub> & B<sub>2</sub> are called "successors" to Block B<sub>0</sub>.

B<sub>0</sub> is called "predecessor" to blocks B<sub>1</sub> & B<sub>2</sub>.

Global optimization :-

Performing transformations or optimizations among multiple blocks is called Global optimization.

The locally optimized blocks are i/p to the Global optimization.

Note :- Means first we should perform local optimization before going to global optimization.

Terms used in Global optimization :-

① point & path :- A point is a place of reference that can be found at

1. before the first statement
2. After the last statement
3. or between two statements within a basic block.

Ex :- In B<sub>0</sub> we have 6 points  
B<sub>1</sub> we have 2 points  
B<sub>2</sub> we have 9 " "  
B<sub>3</sub> we have 5 " "  
B<sub>4</sub> we have 3 " "

Path : A path is a sequence of points in which the control can flow between two points.

Ex:- The path b/w two points  $P_1 \cdot b_0$  &  $P_{12} \cdot b_2$  is the sequence of points :

$P_0 \cdot b_0, P_1 \cdot b_0, P_2 \cdot b_0, P_3 \cdot b_0, P_4 \cdot b_0, P_5 \cdot b_0, P_6 \cdot b_0,$   
 $P_7 \cdot b_0, P_8 \cdot b_0, P_9 \cdot b_0, P_{10} \cdot b_0, P_{11} \cdot b_0, P_{12} \cdot b_2.$

Ex:- The path b/w two points  $P_0 \cdot B_0$  &  $P_{20} \cdot B_2$  is given as

one sequence of points  $P_0 \cdot B_0, P_1 \cdot b_0, P_2 \cdot b_0, P_3 \cdot b_0, P_4 \cdot b_0, P_5 \cdot P_0,$   
 $P_6 \cdot b_0, P_7 \cdot B_1, P_8 \cdot B_1, P_{18} \cdot B_3, P_{19} \cdot B_3, P_{20} \cdot B_3.$

(21)  
 Another sequence of points

$P_1 \cdot b_0, \dots, P_6 \cdot b_0, P_9 \cdot B_2, \dots, P_{17} \cdot B_2, P_{18} \cdot B_3, P_{19} \cdot B_3$   
 &  $P_{20} \cdot B_3.$

Ex:- path b/w  $P_{15} \cdot B_2$  &  $P_8 \cdot B_1.$

There is no sequence of points that can trace control from  $P_{15} \cdot B_2$  to  $P_8 \cdot B_1.$

So, we can say that there is no path b/w

$P_8 \cdot B_1$  &  $P_{15} \cdot B_2.$

## Defining & usage of variable :-

→ In our example ~~we~~ if we take TAC statement

$$x = 2 + a ;$$

Here, we are using the variable 'a'.

And we are defining the variable 'x'.

## Data flow property :-

1. Available Expressions
2. Reaching Definition.
3. Liveness

The process of obtaining information about data flow properties by analysing the input data is called as

"Data flow Analysis".

The data flow properties are computed by using some equations known as "data flow equations".

## Available Expression :-

An Expression "a+b" is available at point 'P' if it satisfy the following conditions.

1. In a flow graph, every path from the initial node to point 'P' evaluates "a+b".
2. After last evaluation of 'a+b' & before reaching point 'P' in every path, there is no subsequent assignment to either 'a' or 'b'.



Then we say the expression 'a+b' is available expression at P.

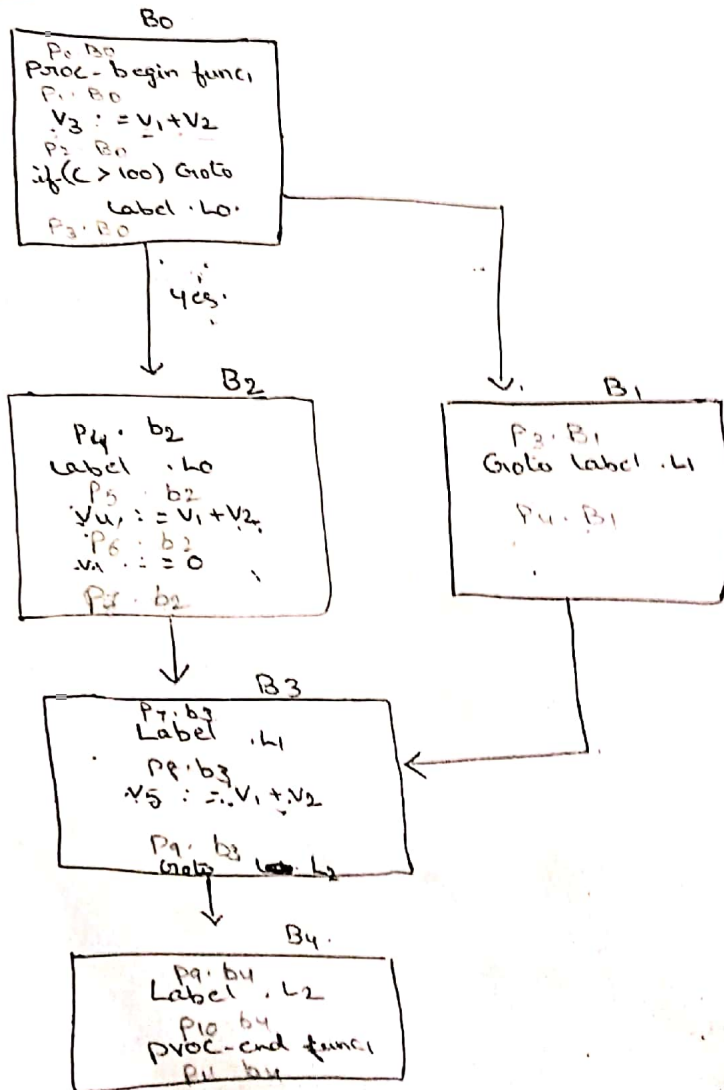
```

Ex:-
int func1 (int c)
{
    v3 = v1 + v2;

    if (c > 100)
    {
        v4 = v1 + v2;
        v1 = 0;
    }
    v5 = v1 + v2;
}

```

Flow Graph :-



This property (AE) of data flow is computed during global optimisation for eliminating re-evaluation of common-sub expressions across blocks.

Data-flow Analysis to compute AE :-

A block generates an expression 'a+b', if it evaluates 'a+b' expression & it does not make any assignment to either 'a' or 'b' variable in that block.

~~That~~ it is denoted as  $e\text{-Gen}[B]$ , where B indicate the block number.

A block kills an expression 'a+b', if it assign a value to either 'a' or 'b' and it doesn't recompute the value of 'a+b' in that block.

It is denoted as  $e\text{-Kill}[B]$ , where B represent the block number in which the expression is killed.

Data flow equations of AE, express the relationship between  $e\text{-in}[B]$  and  $e\text{-out}[B]$  where.

$e\text{-in}[B]$  is the set of expressions that are available at the beginning of the block.

$e\text{-out}[B]$  is the set of expressions that are available at the end of the block.

Data flow equations of  $\Delta E$  :-

$$e\text{-out}[B] = e\text{-Gen}[B] \cup [e\text{-in}[B] - e\text{-kill}[B]]$$

$$e\text{-in}[B] = \bigwedge e\text{-out}[P] \rightarrow \text{for all predecessors } P \text{ of the block}$$

$$e\text{-in}[B_0] = \phi, \text{ if } B_0 \text{ is the initial block.}$$

<u><math>B_0</math></u>	$e\text{-Gen}[B]$	$e\text{-kill}[B]$	$e\text{-in}[B]$
proc-begin func1	$\{v_1 + v_2\}$	$\phi$	$\phi$
$v_3 = v_1 + v_2$			

if  $c > 100$  goto  $t_0$ .

$e\text{-Gen}[B] = \{v_1 + v_2\}$  - This block  $B_0$  is evaluating  $v_1 + v_2$  expression and also there is no assignment to either  $v_1$  or  $v_2$  in this block  $B_0$ .



$e\text{-In}[B_0] = \emptyset$ , Since  $B_0$  is the initial block.

$e\text{-Kill}[B_0] = \emptyset$ , In this block there is no assignment to either  $v_1$  or  $v_2$  variables.

$$\begin{aligned} \therefore e\text{-out}[B_0] &= e\text{-Gen}[B_0] \cup [e\text{-In}[B_0] - e\text{Kill}[B_0]] \\ &= \{v_1 + v_2\} \cup [\emptyset - \emptyset] \\ &= \{v_1 + v_2\}. \end{aligned}$$

$B_1$	$e\text{-Gen}[B_1]$	$e\text{-Kill}[B_1]$	<del><math>e\text{-Kill}</math></del> $e\text{-In}[B_1]$
Goto $\cdot 4$	$\emptyset$	$\emptyset$	$\{v_1 + v_2\}$
	[since this block not evaluating any expression]	[since, it is assigning a value to any variable $v_1$ or $v_2$ ].	

$$\begin{aligned} e\text{-In}[B_1] &= e\text{-out}[B_0] \rightarrow B_0 \text{ is the only predecessor to } B_1 \text{ block.} \\ &= \{v_1 + v_2\}. \end{aligned}$$

$$\begin{aligned} \therefore e\text{-out}[B_1] &= e\text{-Gen}[B_1] \cup [e\text{-In}[B_1] - e\text{Kill}[B_1]] \\ &= \emptyset \cup [\{v_1 + v_2\} - \emptyset] \\ &= \{v_1 + v_2\}. \end{aligned}$$

B<sub>2</sub>

e-Gen[B<sub>2</sub>]

e-Kill[B<sub>2</sub>]

e-In[B<sub>2</sub>]

Label .L<sub>0</sub>

v<sub>4</sub> := v<sub>1</sub> + v<sub>2</sub>

v<sub>1</sub> := 0

∅

[It is assigning value to variable v<sub>1</sub> in the block.]

{v<sub>1</sub> + v<sub>2</sub>}

→ It is assigning value to v<sub>1</sub> & not recomputing the exp. in the block.]

{v<sub>1</sub> + v<sub>2</sub>}

$$e-In[B_2] = e-Out[B_0] - \text{Since 'B}_0\text{' is the only predecessor to B}_2.$$

$$= \{v_1 + v_2\}$$

$$e-out[B_2] = e-Gen[B_2] \cup [e-In[B_2] - e-Kill[B_2]]$$

$$= \emptyset \cup [\{v_1 + v_2\} - \{v_1 + v_2\}]$$

$$= \emptyset \cup \{\emptyset\}$$

$$= \emptyset$$

B<sub>3</sub>

e-Gen[B<sub>3</sub>]

e-Kill[B<sub>3</sub>]

e-In[B<sub>3</sub>]

Label .L<sub>1</sub>

v<sub>5</sub> := v<sub>1</sub> + v<sub>2</sub>

{v<sub>1</sub> + v<sub>2</sub>}

∅

∅

It is. [This block is evaluating v<sub>1</sub> + v<sub>2</sub> & also it doesn't assign value to v<sub>1</sub> & v<sub>2</sub>].

[It is not assigning & doesn't recomputing].

$$e-In[B_3] = e-out[B_1] \wedge \{e-out[B_2] - \text{For B}_3, B_1 \& B_2 \text{ both are predecessors}\}$$

$$= \{v_1 + v_2\} \wedge \emptyset$$

$$= \emptyset$$

$$\begin{aligned}
 e\text{-out}[B_3] &= e\text{-Gen}[B_3] \cup [e\text{-In}[B_3] - e\text{-kill}[B_3]] \\
 &= \{v_1 + v_2\} \cup [\emptyset - \emptyset] \\
 &= \{v_1 + v_2\}.
 \end{aligned}$$

<u>B<sub>4</sub></u>	e-Gen[B <sub>4</sub> ]	e-kill[B <sub>4</sub> ]	e-In[B <sub>4</sub> ]
Label: l2	$\emptyset$	$\emptyset$	$\{v_1 + v_2\}$
proc-end func1			

$$\begin{aligned}
 e\text{-In}[B_4] &= e\text{-out}[B_3] \rightarrow \text{only } B_3 \text{ is predecessor to } B_4 \\
 &= \{v_1 + v_2\}
 \end{aligned}$$

$$\begin{aligned}
 e\text{-out}[B_4] &= e\text{-Gen}[B_4] \cup [e\text{-In}[B_4] - e\text{-kill}[B_4]] \\
 &= \emptyset \cup [\{v_1 + v_2\} - \emptyset] \\
 &= \{v_1 + v_2\}.
 \end{aligned}$$

Block	e-Gen[B]	e-kill[B]	e-In[B]	e-out[B]
B <sub>0</sub>	$\{v_1 + v_2\}$	$\emptyset$	$\emptyset$	$\{v_1 + v_2\}$
B <sub>1</sub>	$\emptyset$	$\emptyset$	$\{v_1 + v_2\}$	$\{v_1 + v_2\}$
B <sub>2</sub>	$\emptyset$	$\{v_1 + v_2\}$	$\{v_1 + v_2\}$	<del><math>\{v_1 + v_2\}</math></del> $\emptyset$
B <sub>3</sub>	$\{v_1 + v_2\}$	$\emptyset$	$\emptyset$	$\{v_1 + v_2\}$
B <sub>4</sub>	$\emptyset$	$\emptyset$	$\{v_1 + v_2\}$	$\{v_1 + v_2\}$



Note :- Data flow properly Available expression

in global optimisation is used to perform

Common - Sub - Expression elimination.

Global Common - Sub expression Elimination using AE :-

Algorithm :-

Identify the statements 'q' that contain an expression of the form  $x := y \text{ op } z$  which satisfy the following

conditions

1.  $P \rightarrow \text{In}[B]$  should contain that expression  $x := y \text{ op } z$ .  
B is the block which contains expression  $x := y \text{ op } z$ .
2. There is no assignment to either y or z variable before 'q' in the block.

The statements 'q' which satisfy these conditions are added to set 'M'.

For each statement 'q' in M do the following to eliminate common sub exp.

1. Identify all the statements evaluating 'y+z' that reach 'q' and add them to set A.
2. Create a new temporary variable 'tn'.
3. For every statement  $w := y \text{ op } z$  in set A,  
 $tn := y \text{ op } z$ .  
 $w := tn$ .
4. Replace 'q' by  $w := tn$ .

In our Example.

In Block  $[B_1]$  :  $e - \text{In}[B_1] = \{v_1 + v_2\}$  And also

There is no assignment to  $v_1$  &  $v_2$  before this stmt.

In Block  $[B_2]$  :  $e - \text{In}[B_2] = \{v_1 + v_2\}$  And also

There is no assignment to  $v_1$  &  $v_2$  before this stmt.

In Block  $[B_4]$  :  $e - \text{In}[B_4] = \{v_1 + v_2\}$  And also

There is no assignment to  $v_1$  &  $v_2$  before this stmt.

$\therefore$  Set  $m = \{B_1, B_2, B_4\} \rightarrow B_1 \ \& \ B_4$  Not evaluating  $v_1 + v_2$ , so

Now, if we take  $B_2$ .  
 $B_0$  is the only block

Set  $A = \{B_2\}$ .  
 $\downarrow$   
quad(5)

① statement that is evaluating  $\{v_1 + v_2\}$  that reach quad(5) in block  $B_2$  is quad(5) in  $B_0$ .

② ~~so~~ creating temporary variable to as

$t_0 := v_1 + v_2;$

$v_3 := t_0.$

③ Replace the quad(5) in  $B_2$  as  $v_4 := t_0.$

$B_0$ . After opti.

proc - begin func

$t_0 := v_1 + v_2$

$v_3 := t_0$

if  $c > 100$  goto .L0.

$B_2$ . After opti

Label .L0

$v_4 := t_0$

$v_1 := 0.$

# Live Variable Analysis

## Input source

```
int func (int a, int b)
{
  int i, j, k;
  i = 45;
  j = a + b;
  // i is replaced by 45 * /
  if ( (a + i) > 100 ) {
    k = a + j;
  }
  else
  {
    k = b + j;
  }
  return(k);
}
```

## TAC after local optimization

B0	(0) proc-begin func (1) i := 45 (2) j = a + b (3) t <sub>1</sub> := a + 45 (4) if t <sub>1</sub> > 100 goto L0
B1	(5) goto L1
B2	(6) label L0 (7) k := a + j (8) goto L2
B3	(9) label L1 (10) k := b + j
B4	(11) label L2 (12) return k (13) goto L3
B5	(14) label L3 (15) proc-end func.

→ A variable  $v$  is said to be live at a point  $p$ , if it is used in some path in the flow graph starting  $p$ .

→ The variable is considered dead, if it is not live.

→ The global dead code elimination is done using live-variable analysis.

- The data flow analysis done for the data flow property liveness is called "live-variable analysis".



→ We use the term line-DEFS[B] to represent the set of variables whose definition precedes any use in the block B.

→ We use the term line-USES[B] to represent the set of variables whose use precedes any definition within the block B.

- The data flow equations for live variable analysis express the relationship between the line-IN[B] & line-OUT[B]

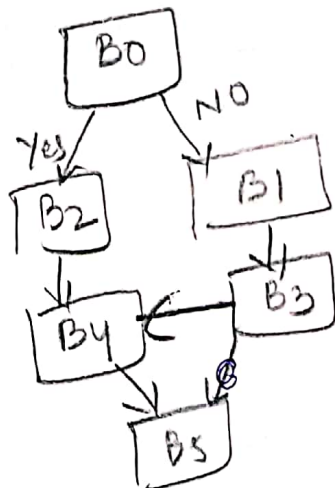
line-IN[B] is the set of all the variables that are live before reaching the beginning of the block 'B'.

line-OUT[B] is the set of variables that are live at the end of block 'B'.

Data flow equations for live variable analysis are

$$\text{line-IN}[B] = \text{line-USES}[B] \cup (\text{line-OUT}[B] - \text{line-DEFS}[B])$$

$$\text{line-OUT}[B] = \bigcup \text{line-IN}[S] \dots, \text{ where } S \text{ is the set of successors of the block } B.$$



The calculations of live-DEFS, live-USES, live-IN &

(2)

TAC after local opt	live-DEFS	live-USES	live-OUT	live-IN
B0 (0) proc - begin func	live-DEFS(B0) = {i, j, t1}	live-USES(B0) = {a, b}	live-OUT(B0) = {a, b, j}	live-IN(B0) = {a, b}
(1) i := 45				
(2) j := a + b				
(3) t1 := a + 45				
(4) if t1 > 100 goto L0				
B1 (5) goto L1	live-USES(B1) = {j}	live-DEFS(B1) = {}	live-OUT(B1) = {b, j}	live-IN(B1) = {b, j}
B2 (6) label L0	live-DEFS(B2) = {k}	live-USES(B2) = {a, j}	live-OUT(B2) = {k}	live-IN(B2) = {a, j}
(7) k := a + j				
(8) goto L2				
B3 (9) label L1	live-DEFS(B3) = {k}	live-USES(B3) = {b, j}	live-OUT(B3) = {k}	live-IN(B3) = {b, j}
(10) k := b + j				
B4 (11) label L2	live-DEFS(B4) = {}	live-USES(B4) = {k}	live-OUT(B4) = {}	live-IN(B4) = {k}
(12) return k				
(13) goto L3				
B5 (14) label L3	live-DEFS(B5) = {}	live-USES(B5) = {}	live-OUT(B5) = {}	live-IN(B5) = {}
(15) proc - end func				

$$\text{line-OUT}(B_5) = \{j\}$$

$$\begin{aligned} \text{line-IN}(B_5) &= \{i\} \cup \{k\} - \{j\} \\ &= \{i\} \end{aligned}$$

$$\text{line-OUT}(B_4) = \text{line-IN}(B_5) = \{i\}$$

$$\text{line-IN}(B_4) = \{k\} \cup \{i\} - \{i\} = \{k\}$$

$$\text{line-OUT}(B_3) = \text{line-IN}(B_4) = \{k\}$$

$$\begin{aligned} \text{line-IN}(B_3) &= \{b, j\} \cup \{k\} - \{k\} \\ &= \{b, j\} \end{aligned}$$

$$\text{line-OUT}(B_2) = \text{line-IN}(B_3) = \{b, j\}$$

$$\begin{aligned} \text{line-IN}(B_2) &= \{a, i\} \cup \{k\} - \{k\} \\ &= \{a, i\} \end{aligned}$$

$$\text{line-OUT}(B_1) = \text{line-IN}(B_2) = \{a, i\}$$

$$\begin{aligned} \text{line-IN}(B_1) &= \{i\} \cup \{b, j\} - \{i\} \\ &= \{b, j\} \end{aligned}$$

$$\text{line-OUT}(B_0) = \text{line-IN}(B_1) \cup \text{line-IN}(B_2) = \{a, b, j\}$$

$$\begin{aligned} \text{line-IN}(B_0) &= \{a, i, b\} \cup \{a, i, b, j\} - \{a, i, j\} \\ &= \{a, i, b\} \cup \{a, i, b\} = \{a, i, b\} \end{aligned}$$



# Dead Code Elimination using Live Variable

(3)

## Analysis Information

Consider the block  $B_0$  & calculate the line-OUT information for each of the quad.

- $B_0$
- (0) proc - begin func
  - (1)  $i := 45$
  - (2)  $j := a + b$
  - (3)  $t_1 := a + 45$
  - (4) if  $t_1 > 100$  goto  $L_0$ .

$$\text{line-OUT}(B_0) = \{a, b, j\}$$

NOTE: The line-OUT of the last quad in the block is the same as the line-OUT of the block.

## Calculating the line-OUT information for each quad.

quad	line-OUT	line-IN
(4) if $t_1 > 100$ goto $L_0$	$\{a, b, j\}$ $\text{line-DEFS}[q_4] = \{j\}$ $\text{line-USES}[q_4] = \{t_1\}$	$\{t_1, a, b, j\}$
(3) $t_1 := a + 45$	$\{t_1, a, b, j\}$ $\text{line-DEFS}[q_3] = \{t_1\}$ $\text{line-USES}[q_3] = \{a, b\}$	$\{a, b, j\}$
(2) $j := a + b$	$\{a, b, j\}$ $\text{line-DEFS}[q_2] = \{j\}$ $\text{line-USES}[q_2] = \{a, b\}$	$\{a, b, j\}$
(1) $i := 45$	$\{a, b, j\}$ $\text{line-DEFS}[q_1] = \{i\}$ $\text{line-USES}[q_1] = \{j\}$	$\{a, b, j\}$

→ live-out information at each quad can be used in the global dead code elimination.

Block #	Quad	live-out
0	proc-begin func	{a, b}
1	i := 45	{a, b}
2	j := a + b	{a, b, j}
3	t1 := a + 45	{a, b, j, t1}
4	if t1 > 100 goto L0	{a, b, j}

Algorithm formalizes the idea of global dead code elimination using the live-out information

for each quad 'q' of the form  $x := y \text{ op } z$  in the block

if (live-out[q] does not contain x) ~~if x is dead~~

if x is dead

eliminate the quad q

end

→ Here in quad #1, the variable 'i' is not a member of live-out. i.e. 'i' is dead  
 ∴ quad #1 can be eliminated

∴ TAC after global dead code elimination is.

- (0) proc-begin func
- (1) j := a + b
- (2) t1 := a + 45
- (3) if t1 > 100 goto L0

# Reaching Definition

- We use the RD property to perform optimizations in loop.

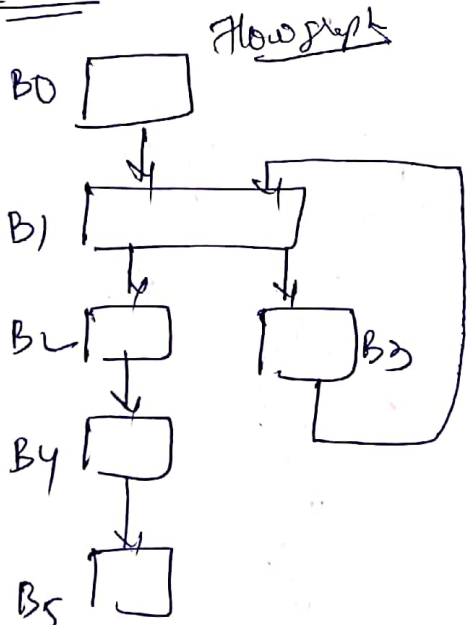
Detection of loop - The loops in programs are detected during the data flow analysis by using a concept called as "domination" in a flow graph

→ Here we examine the concepts & algorithms to:

- (a) Detect the presence of a loop, given the intermediate code.
- (b) Identify the basic blocks in the intermediate code that constitute a loop.

→ A node 'd' of a flow graph dominates node 'n', if every path from the initial to 'n' goes through 'd'. It is represented as 'd dom n'

Note :- Each node dominates itself.



## Dominators

- dominators[0] = {0}
- [1] = {0, 1}
- [2] = {0, 1, 2}
- [3] = {0, 1, 3}
- [4] = {0, 1, 2, 4}
- [5] = {0, 1, 2, 4, 5}

→ For an edge in a flow graph  $a \rightarrow b$   
↓ ↓  
tail head

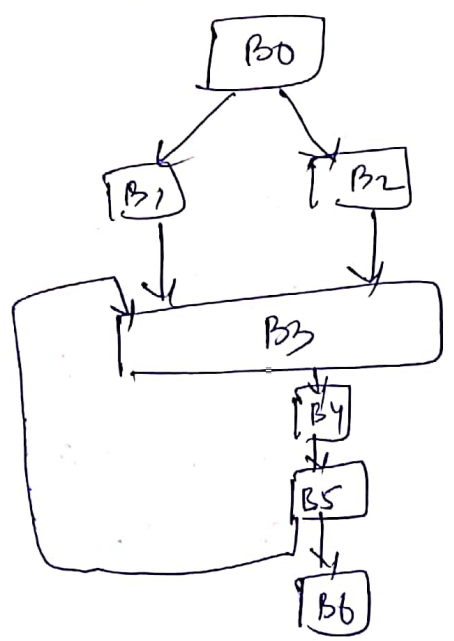


## Edges & dominators for head & tail

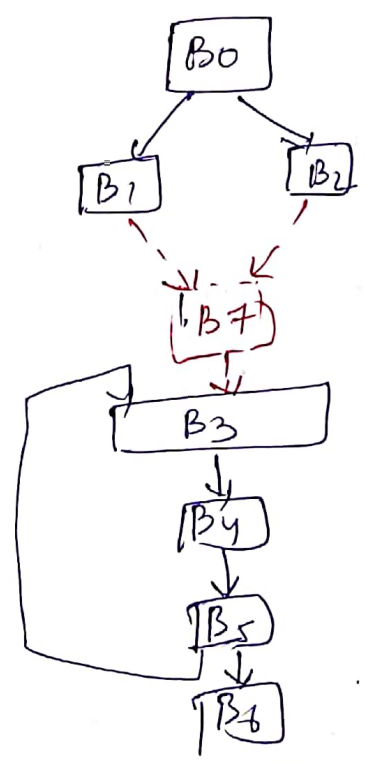
Edge	Head	Tail	dominator [head]	dom[tail]	Remarks
0 → 1	1	0	{0, 1}	{0}	
1 → 2	2	1	{0, 1, 2}	{0, 1}	
2 → 4	4	2	{0, 1, 2, 4}	{0, 1, 2}	
4 → 5	5	4	{0, 1, 2, 4, 5}	{0, 1, 2, 4}	
1 → 3	3	1	{0, 1, 3}	{0, 1}	
3 → 1	1	3	{0, 1}	{0, 1, 3}	Back edge

→ If dominator [tail] contain the head, then the edge is called a back edge.

→ Presence of a back edge indicates the existence of a loop in a flow graph.



Before the introduction of preheader



After the introduction of preheader

- In some of the loop optimization techniques, say code motion, it is required to move several quads from within the loop to outside of the loop.
- In the optimized code, these quads would typically need to be executed before entering the loop.
- A pre-header block serves as a placeholder for the quads that need to be executed just before entering the loop.
- The pre-header is a basic block introduced during the loop optimization to hold the quads that are moved from within the loop.
- It is a predecessor to the header block.

Identifying the basic blocks forming a loop

- Given a back edge  $n_1 \rightarrow n_2$ , a natural loop is  $n_1, n_2$  & the set of nodes that can reach  $n_1$  without going through  $n_2$ .
- We apply reachability definition ~~to~~ in one of the loop optimization called as loop invariant code motion

# Input source

```

int arr[1000];
int func(int a, int b)
{
    int i;
    int n1, n2;
    i = 0;
    n1 = a * b;
    n2 = a - b;
    while (arr[i] > (n1 * n2))
    {
        i = i + 1;
    }
    return(i);
}

```

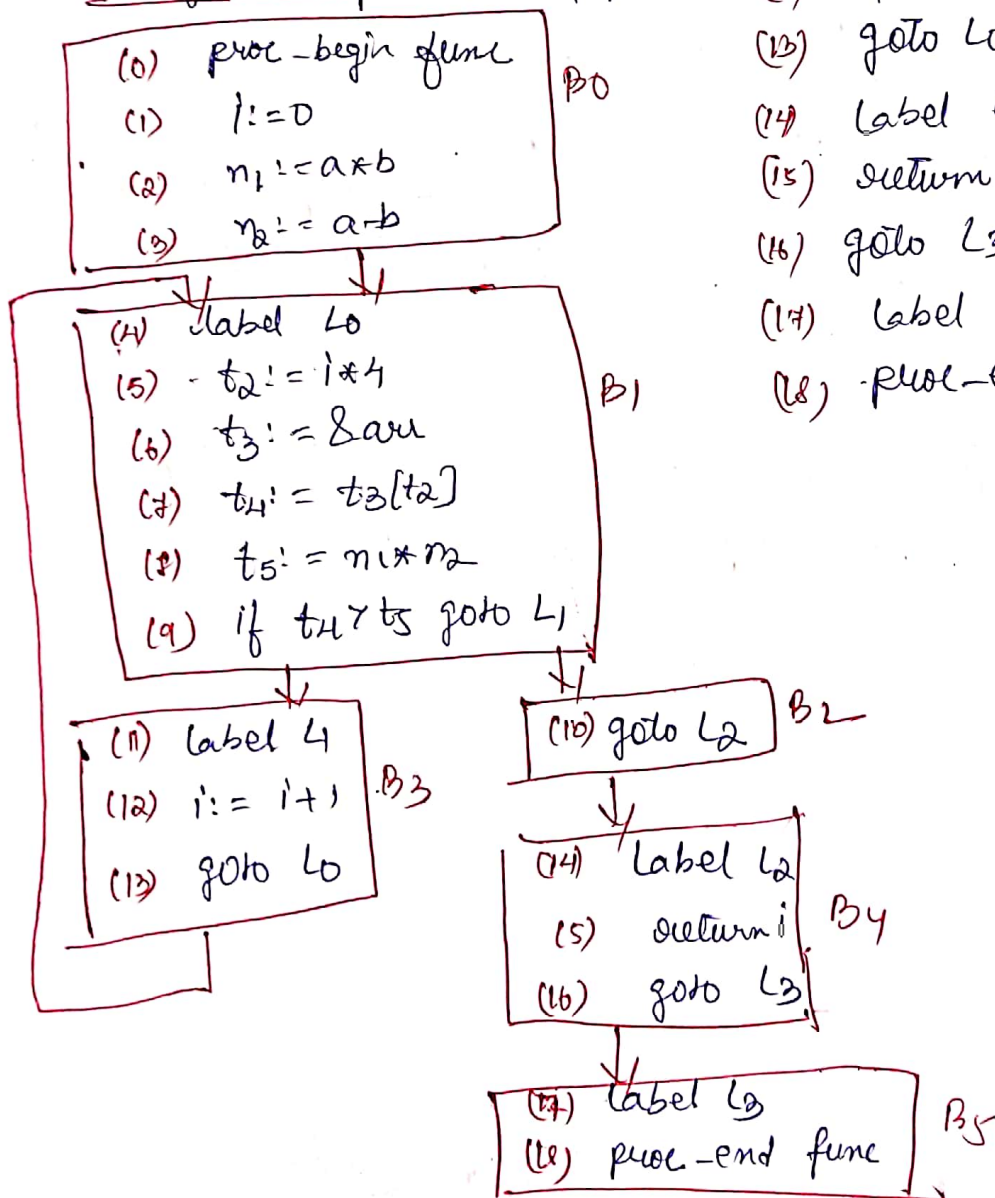
# Locally optimized TAC

```

(0) proc-begin func
(1) i := 0
(2) n1 := a * b
(3) n2 := a - b
(4) Label L0
(5) t2 := i * 4
(6) t3 := &arr
(7) t4 := t3[t2]
(8) t5 := n1 * n2
(9) if t4 > t5 goto L1
(10) goto L2
(11) Label L1
(12) i := i + 1
(13) goto L0
(14) Label L2
(15) return i
(16) goto L3
(17) Label L3
(18) proc-end func

```

# TAC after local opt & flow graph





- To understand the terminology that is required for computing the reaching definitions  $\rightarrow$

The universal set of definitions  $L$  consists of any definition that appears in the statements.

- Here definitions can be seen in the quads 1, 2, 3, 5, 6, 7, 8, 11, 12

$$\therefore L = \{1, 2, 3, 5, 6, 7, 8, 12\}$$

$\rightarrow$  A block generates a definition 'd', if the definition made reaches the end of the block. We use the term rd-GEN[B] to denote the set of definitions generated by a basic block B.

$\rightarrow$  A block kills all the definitions of a variable 'x' made outside the block, if it assigns a value to 'x'. We use the term rd-KILL[B] to denote the definitions killed by a basic block B.

The data flow equations are

$$\text{rd-OUT}[B] = \text{rd-GEN}[B] \cup (\text{rd-IN}[B] - \text{rd-KILL}[B])$$

$$\text{rd-IN}[B] = \bigcup \text{rd-OUT}[P], \text{ for all the predecessors of } P \text{ of the block.}$$

#	TAC	$\sigma d\_GEN$	$\sigma d\_IN$ & $\sigma d\_OUT$
B0	(0) proc-begin func (1) $i := 0$ (2) $m1 := a \times b$ (3) $m2 := a - b$	$\sigma d\_GEN[B0] = \{1, 2, 3\}$ $\sigma d\_KILL[B0] = \{1, 2\}$	$\sigma d\_IN[B0] = \{1\}$ $\sigma d\_OUT[B0] = \{1, 2, 3\}$
B1	(4) Label L0 (5) $t2 := 1 \times 4$ (6) $t3 := 2 \times m1$ (7) $t4 := t3(t2)$ (8) $t5 := m1 \times m2$ (9) If $t4 > t5$ goto L1	$\sigma d\_GEN[B1] = \{5, 6, 7, 8\}$ $\sigma d\_KILL[B1] = \{1\}$	$\sigma d\_IN[B1] = \{1, 2, 3, 11, 12\}$ $\sigma d\_OUT[B1] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$
B2	(10) goto L2	$\sigma d\_GEN[B2] = \{1\}$ $\sigma d\_KILL[B2] = \{1\}$	$\sigma d\_IN[B2] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$ $\sigma d\_OUT[B2] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$
B3	(11) Label L1 (12) $i := i + 1$ (13) goto L0	$\sigma d\_GEN[B3] = \{1, 2\}$ $\sigma d\_KILL[B3] = \{1, 2\}$	$\sigma d\_IN[B3] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$ $\sigma d\_OUT[B3] = \{2, 3, 5, 6, 7, 8, 11, 12\}$
B4	(14) Label L2 (15) return i (16) goto L3	$\sigma d\_GEN[B4] = \{1\}$ $\sigma d\_KILL[B4] = \{1\}$	$\sigma d\_IN[B4] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$ $\sigma d\_OUT[B4] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$
B5	(17) Label L3 (18) proc-end func	$\sigma d\_GEN[B5] = \{1\}$ $\sigma d\_KILL[B5] = \{1\}$	$\sigma d\_IN[B5] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$ $\sigma d\_OUT[B5] = \{1, 2, 3, 5, 6, 7, 8, 11, 12\}$

{Initial value of  $\sigma d\_OUT(B)$  for empty block B is assumed as  $\sigma d\_GEN(B)$ }

→ with these values repeat the whole process as second iteration.

## Loop Invariant Code Motion Optimization using RD Analysis

There are two steps required for performing the loop invariant code motion optimization. They are:

- 1) The deletion of loop invariant statements in the loop.  
This is based on the ud-chain inf. obtained from the reaching definition analysis.
- 2) The moving of the loop invariant stmts to the pre-header of the loop.

### Detection of loop invariant statements

A stmt ' $s: x = y + z$ ' in a loop  $L$  is considered as loop invariant if one of the following conditions hold good:

- 1) All the reaching definitions of ' $y$ ' & ' $z$ ' at ' $s$ ' are from outside the loop as indicated by the ud chains for the quad  $q$ .
- 2) The operands ' $y$ ' & ' $z$ ' are constants.

### ② Moving the Loop Invariant Statements to the Pre-header

For a stmt ' $s: a = b * c$ ', to be moved into the pre-header, the following are the conditions that should be met.

- 1) There should be no other stmt ' $s$ ' which defines ' $a$ ' within  $L$ .
- 2) The reaching definition for all the uses of ' $a$ ' in the loop should be from ' $s$ ' only.
- 3) The stmt ' $s$ ' should be in a block that dominates all the exits of the loop  $L$ .



2<sup>nd</sup> iteration

#	TAC	old-GEN & old-KILL	old-IN & old-OUT
	(0) proc - begin func	old-GEN[B0] = $\{1, 2, 3\}$	old-IN[B0] = $\{1\}$
B0(1)	i := 0	old-KILL[B0] = $\{1, 2, 3\}$	old-OUT[B0] = $\{1, 2, 3\}$
	(2) $n_1 := a * b$		
	(3) $n_2 := a - b$		
	(4) Label L0	old-GEN[B1] = $\{5, 6, 7, 8\}$	old-IN[B1] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
	(5) $t_2 := i * 4$	old-KILL[B1] = $\{1\}$	
B1(6)	$t_3 := 2 * n_1$		old-OUT[B1] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
(7)	$t_4 := t_3(t_2)$		
	(8) $t_5 := n_1 * n_2$		
	(9) if $t_4 > t_5$ goto L1		
B2(10)	goto L2	old-GEN[B2] = $\{1\}$	old-IN[B2] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
		old-KILL[B2] = $\{1\}$	old-OUT[B2] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
B3(11)	Label L1	old-GEN[B3] = $\{1, 2\}$	old-IN[B3] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
(12)	i := i + 1	old-KILL[B3] = $\{1\}$	old-OUT[B3] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
(13)	goto L0		
B4(14)	Label L2	old-GEN[B4] = $\{1\}$	old-IN[B4] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
(15)	return i	old-KILL[B4] = $\{1\}$	old-OUT[B4] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
(16)	goto L3		
B5(17)	Label L3	old-GEN[B5] = $\{1\}$	old-IN[B5] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$
(18)	proc end func	old-KILL[B5] = $\{1\}$	old-OUT[B5] = $\{1, 2, 3, 5, 6, 7, 8, 12\}$

UD-OUT in the first & 2nd iteration :

Block #	UD-OUT in the iteration #1	UD-OUT in the iteration #2
0	UD-OUT[B0] = {1, 2, 3}	# same.
1	UD-OUT[B1] = {1, 2, 3, 5, 6, 7, 8, 12}	
2	UD-OUT[B2] = {1, 2, 3, 5, 6, 7, 8, 12}	
3	UD-OUT[B3] = {2, 3, 5, 6, 7, 8, 10}	
4	UD-OUT[B4] = {1, 2, 3, 5, 6, 7, 8, 12}	
5	UD-OUT[B5] = {1, 2, 3, 5, 6, 7, 8, 12}	

UD-chain (use-definition)

It is a set holding all definitions reaching a quad, for each variable used in the quad.

UD-chain(8, n1) = {2}

UD-chain(8, n2) = {3}

UD-chain Information

Block	Quad	UD-chain Information	Explanation
B1	(5) t2 := i * 4	UD-chain(5, i) = {1, 12}	
B1	(6) t3 := 2 * t2	None	
B1	(7) t4 := t3 [t2]	UD-chain(7, t3) = {6} UD-chain(7, t2) = {5}	
B1	(8) t5 := n1 * n2	UD-chain(8, n1) = {2} ✓ UD-chain(8, n2) = {3}	
B1	(9) if t4 > t5 goto L1	UD-chain(9, t4) = {7} UD-chain(9, t5) = {8}	